Theoretical and Mathematical Physics, **181**(2): 1396–1404 (2014)

CORRELATED LLOYD MODEL: EXACT SOLUTION

© **G. G. Kozlov***

We describe an exactly solvable model of a disordered system that is a generalized Lloyd model; it differs from the classical model because the random potential is not a δ-correlated random process. We show that the exact average Green's function in this case is independent of the correlation radius of the random potential and, as in the classical Lloyd model, is a crystal Green's function whose energy argument acquires an imaginary part dependent on the disorder degree.

Keywords: Lloyd model, exactly solvable model, correlated disordered system, density of states, average Green's function

1. Introduction

The number of published papers dedicated to models of disordered systems has noticeably increased recently; the study of systems with *correlated* disorder [1]–[11] becomes more and more popular. In the historically first models of random systems, a δ-correlated random potential was typically studied, and relatively little attention was focused on the problem of the influence of correlations. Recent studies showed that correlations can lead to significant (sometimes qualitative) changes in the energy structure and the localization properties of disordered systems.

Exactly solvable models, whose number is usually small, play an extremely important role in any field of theoretical physics. An exactly solvable model can allow accumulating qualitative information on the class of models to which it belongs with a high degree of reliability and permits verifying applied approximations and indicating the direction for further studies. In the physics of disordered systems, the exactly solvable Dyson [12] and Lloyd [13] models related to noncorrelated disordered systems are the best known and most important. Here, we present an exact calculation of the average Green's function for a generalized one-dimensional Lloyd model in which *site energies are not independent random quantities* and show that it is independent of the model parameter playing the role of the correlation radius.

This paper is organized as follows. In Sec. 2, we consider the classical Lloyd model and give needed results and definitions. In Sec. 3, we describe correlated disorder, for which we give the exact calculation of the average Green's function in Sec. 4.

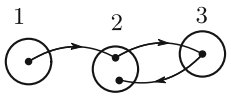
2. Lloyd model

The matrix of the Hamiltonian **H** of the classical one-dimensional Lloyd model has the elements

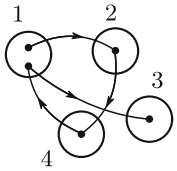
$$H_{rr'} = \delta_{rr'}\varepsilon_r + w(r - r'), \quad r, r' = 1, 2, \ldots, N. \tag{1}$$

*Fock Institute of Physics, St. Petersburg State University, St. Petersburg, Russia,
e-mail: gkozlov@photonics.phys.spbu.ru.

Translated from Teoreticheskaya i Matematicheskaya Fizika, Vol. 181, No. 2, pp. 312–321, November, 2014. Original article submitted March 17, 2014.



$$= \frac{1}{E-\varepsilon_1} w_{12} \frac{1}{E-\varepsilon_2} w_{23} \frac{1}{E-\varepsilon_3} w_{32} \frac{1}{E-\varepsilon_2}$$



$$= \frac{1}{E-\varepsilon_1} w_{12} \frac{1}{E-\varepsilon_2} w_{24} \frac{1}{E-\varepsilon_4} w_{41} \frac{1}{E-\varepsilon_1} w_{13} \frac{1}{E-\varepsilon_3}$$

Fig. 1. Examples of diagrams.

In this case, the function $w(r)$ is assumed to be given and such that $w(0) = 0$, the diagonal elements (site energies) ε_r are independent and equally distributed random quantities, and their distribution density has the form (Cauchy distribution)

$$P(\varepsilon) = \frac{1}{\pi} \frac{\Delta}{\Delta^2 + \varepsilon^2}. \tag{2}$$

The parameter Δ characterizes the degree of system disorder: as $\Delta \to 0$, Hamiltonian (1) corresponds to the ordered (crystal) system and is diagonalized in the plane-wave representation. The set of numbered site energies is often called a random potential.

Lloyd [13] managed to exactly calculate the matrix of the average Green's function $\langle \mathbf{G}(\Omega) \rangle = \langle [\Omega - \mathbf{H}]^{-1} \rangle$, $\Omega \equiv E + i\delta$, $\delta \to +0$, for model system (1), (2). It determines the spectrum of the linear susceptibility and the density of system states. In this section, we reproduce the Lloyd result using the diagram technique presented in detail in [14] (also see [15]) and described as follows. We introduce the matrix $\mathbf{W}$ with the elements $W_{rr'} \equiv w(r - r')$. The matrix of the Green's function $\mathbf{G}$ can then be represented in the form of the series [16]

$$G_{rr'}(\Omega) = \frac{\delta_{rr'}}{\Omega - \varepsilon_r} + \frac{1}{\Omega - \varepsilon_r} W_{rr'} \frac{1}{\Omega - \varepsilon_{r'}} + \sum_{r''} \frac{1}{\Omega - \varepsilon_r} W_{rr''} \frac{1}{\Omega - \varepsilon_{r''}} W_{r''r'} \frac{1}{\Omega - \varepsilon_{r'}} + \ldots. \tag{3}$$

We set the quantities $W_{rr'}$ into correspondence with the arrow connecting the sites r and r' and set each multiplier $[\Omega - \varepsilon_r]^{-1}$ into correspondence with the bold point at the corresponding site r, which we represent by a circle. We give examples in Fig. 1.

For the matrix element $G_{rr'}(\Omega)$ of the Green's function, we can then write the expression

$$G_{rr'}(\Omega) = \text{the sum of all diagrams connecting the sites } r \text{ and } r'. \tag{4}$$

To calculate the sought average Green's function, we should integrate (4) with the joint distribution function of the site energies $\varepsilon_1, \ldots, \varepsilon_N$, which is given by the product

$$\rho_{\text{nc}}(z_1, z_2, \ldots, z_N) = \prod_{j=1}^{N} P(z_j) \tag{5}$$

in the case of noncorrelated disorder; in this case, the function $P(z)$ for the considered Lloyd model has form (2). Averaging the arbitrary diagram D in expansion (4) involves averaging the factor f_D defined as

$$f_D \equiv \left(\frac{1}{\Omega - \varepsilon_{n_1}} \right)^{g_1} \left(\frac{1}{\Omega - \varepsilon_{n_2}} \right)^{g_2} \cdots \left(\frac{1}{\Omega - \varepsilon_{n_q}} \right)^{g_q}, \tag{6}$$

where $n_1, n_2, \ldots, n_q$ are the numbers of sites through which the diagram D passes and $g_1, g_2, \ldots, g_q$ are the numbers of passages of the diagram through the respective sites $n_1, n_2, \ldots, n_q$ (g_i is the number of bold points at the site n_i). For example, for the upper diagram in Fig. 1, $n_1 = 1$, $n_2 = 2$, $n_3 = 3$ and $g_1 = 1$, $g_2 = 2$, $g_3 = 1$. The expression

$$\int P(z)\left(\frac{1}{E + i0 - z}\right)^n dz = \frac{1}{\pi}\int \frac{\Delta\, dz}{\Delta^2 + z^2}\left(\frac{1}{E + i0 - z}\right)^n = \left(\frac{1}{E + i\Delta}\right)^n \tag{7}$$

is key in the Lloyd solution. Taking this relation into account and using the mutual independence of the random site energies ε_{n_j}, $j = 1, 2, \ldots, q$, we obtain the expression for the average factor f_D:

$$\langle f_D\rangle = \int \prod_{j=1}^{N} dz_j P(z_j)\left(\frac{1}{\Omega - z_{n_1}}\right)^{g_1}\left(\frac{1}{\Omega - z_{n_2}}\right)^{g_2}\cdots\left(\frac{1}{\Omega - z_{n_q}}\right)^{g_q} =$$

$$= \left(\frac{1}{\Omega + i\Delta}\right)^{g_1}\left(\frac{1}{\Omega + i\Delta}\right)^{g_2}\cdots\left(\frac{1}{\Omega + i\Delta}\right)^{g_q}. \tag{8}$$

This expression corresponds to the diagram D of the Green's function of the *ordered* system all of whose site energies are zero and for which the energy argument is replaced as $\Omega \to \Omega + i\Delta$. Because this calculation holds for any diagram in expression (4), we obtain Lloyd's result: *the average Green's function of the disordered system with Hamiltonian* (1) *and uncorrelated disorder of the Cauchy type* (2) *is equal to the Green's function* $\mathbf{G}^{\mathrm{od}}$ *of the ordered system with the Hamiltonian of form* (1) *for* $\varepsilon_r = 0$, $r = 1, 2, \ldots, N$, *in which the energy argument must be replaced as* $\Omega \to \Omega + i\Delta$:

$$\langle \mathbf{G}(\Omega)\rangle = \mathbf{G}^{\mathrm{od}}(\Omega + i\Delta). \tag{9}$$

The explicit form of the matrix of the Green's function $\mathbf{G}^{\mathrm{od}}$ of the ordered (and cyclic) system can be written [16] using the fact that eigenvectors of (1) for $\varepsilon_r = 0$, $r = 1, 2, \ldots, N$, are plane waves. For the one-dimensional system, this matrix has the form

$$G^{\mathrm{od}}_{rr'}(\Omega) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{e^{i(r-r')q}}{\Omega - J_q}\, dq, \qquad J_q = \sum_r w(r)e^{-iqr}. \tag{10}$$

In the next section, we describe the correlated discrete random process $\varepsilon_1, \ldots, \varepsilon_i, \ldots, \varepsilon_N$, whose total joint distribution function $\rho(z_1, z_2, \ldots, z_N)$ is not represented by a product of form (5). In Sec. 4, we show that result (9) is retained for the disordered system with Hamiltonian (1) in which the site energies ε_r are an implementation of such a random process.

3. Correlated disorder

We obtain the correlated sequence of site energies ε_r, $r = 1, 2, \ldots, N$, using the following *smoothing* procedure[1] [17], [18]. We introduce a set of independent random quantities ξ_i, $i = -\infty, \ldots, -1, 0, 1, \ldots, +\infty$. Let each of the introduced random quantities ξ_i have the distribution function $P(\xi)$, which we assume to be given and to be independent of i. We obtain the site energies ε_n from these quantities as the implementation of the discrete random process

$$\varepsilon_n = (1 - e^{-\alpha})\sum_{m\le n} e^{\alpha(m-n)}\xi_m, \quad \alpha > 0, \quad n = 1, 2, \ldots, N. \tag{11}$$

[1] A similar correlation mechanism was considered in [10] when calculating the degree of state localization in a correlated system.

For such a construction, the ε_n are correlated with the characteristic correlation radius $R = 1/\alpha$. In this case, for the correlation function $\langle \varepsilon_n \varepsilon_{n'} \rangle$, we have the expression

$$\langle \varepsilon_n \varepsilon_{n'} \rangle = \langle \xi^2 \rangle \left(\frac{1 - e^{-\alpha}}{1 + e^{-\alpha}} \right) e^{-\alpha |n - n'|} = \langle \xi^2 \rangle \left(\frac{1 - \beta}{1 + \beta} \right) e^{-\alpha |n - n'|}, \tag{12}$$

where $\beta \equiv e^{-\alpha}$, $\beta \in [0, 1]$.

From definition (11), it is easy to obtain the relation

$$\varepsilon_{n+1} = \beta \varepsilon_n + (1 - \beta) \xi_{n+1}, \tag{13}$$

which is important for the further study. Correlation function (12) is meaningful only for the finite second moment $\langle \xi^2 \rangle$ of the function $P(\xi)$ and does not exist in the case where $P(\xi)$ is Cauchy function (2). Nevertheless, in that case, sequence (11) also cannot be regarded as uncorrelated because its distribution function has no form (5), as is seen below. Finally, we note that sequence (11) is causally correlated, i.e., ε_n only depends on ξ_m for which $m \le n$.

3.1. Distribution function of the site energy in the correlated chain. The distribution function of any site energy ε_n of random process (11) is independent of its number n. We let $\sigma(\varepsilon)$ denote the indicated function and calculate it for $n = 0$ using the standard method [17], [18]. We initially use the following general expressions for the sought function $\sigma(\varepsilon)$ and the corresponding characteristic function $\tilde{\sigma}(t)$:

$$\sigma(\varepsilon) = \left\langle \delta \left(\varepsilon - [1 - \beta] \sum_{m=0}^{\infty} \beta^m \xi_m \right) \right\rangle \equiv \int e^{i\varepsilon t} \tilde{\sigma}(t), \tag{14}$$

$$\tilde{\sigma}(t) = \frac{1}{2\pi} \left\langle \exp \left[-it \left([1 - \beta] \sum_{m=0}^{\infty} \beta^m \xi_m \right) \right] \right\rangle = \frac{1}{2\pi} \prod_{m=0}^{\infty} \int P(\xi) e^{-it[1-\beta]\beta^m \xi} \, d\xi. \tag{15}$$

Here, the angle brackets correspond to averaging over the independent random quantities ξ_m. Letting $\widetilde{P}(t)$ denote the Fourier transform of $P(\xi)$,

$$\widetilde{P}(t) = \int P(\xi) e^{-it\xi} \, d\xi,$$

we obtain the formula for $\tilde{\sigma}(t)$:

$$\tilde{\sigma}(t) = \frac{1}{2\pi} \prod_{m=0}^{\infty} \widetilde{P}(t[1 - \beta]\beta^m). \tag{16}$$

If $P(\xi)$ is Cauchy distribution (2), then

$$\widetilde{P}(t) = e^{-|t\Delta|} \quad \Rightarrow \quad \tilde{\sigma}(t) = \frac{1}{2\pi} \exp \left(-|t|[1 - \beta]\Delta \sum_{m=0}^{\infty} \beta^m \right) = \frac{1}{2\pi} e^{-|t|\Delta}, \tag{17}$$

and we conclude that the distribution function of site energies in the form of random process (11) also has the form of Cauchy function (2) in this case:

$$\sigma(\varepsilon) = P(\varepsilon) = \frac{1}{\pi} \frac{\Delta}{\Delta^2 + \varepsilon^2}. \tag{18}$$

3.2. Distribution function of random process (11). Discrete correlated random process ε_n (11) is completely determined by the joint distribution function of all site energies $\rho(z_1, z_2, \ldots, z_N)$, which can be calculated analytically. For this, we introduce functions of the joint probability density of the first M $(0 < M \le N)$ site energies $\rho_M(z_1, z_2, \ldots, z_M)$, and $\rho_M(z_1, z_2, \ldots, z_M)\, dz_1 \cdots dz_M$ is hence the probability that $\varepsilon_i \in [z_i, z_i + dz_i]$, $i = 1, 2, \ldots, M$. If relation (13) is used, then we can obtain a recurrence relation expressing ρ_{M+1} in terms of ρ_M:

$$\begin{aligned}
\rho_{M+1}(z_1, z_2, \ldots, z_{M+1}) &= \langle \delta(z_1 - \varepsilon_1) \cdots \delta(z_M - \varepsilon_M) \delta(z_{M+1} - \varepsilon_{M+1}) \rangle = \\
&= \langle \delta(z_1 - \varepsilon_1) \cdots \delta(z_M - \varepsilon_M) \delta(z_{M+1} - \beta\varepsilon_M - [1-\beta]\xi_{M+1}) \rangle = \\
&= \int d\xi\, dy_1 \cdots dy_M\, \rho_M(y_1, \ldots, y_M) \times \\
&\quad \times P(\xi) \delta(z_1 - y_1) \cdots \delta(z_M - y_M) \delta(z_{M+1} - \beta y_M - [1-\beta]\xi) = \\
&= \frac{1}{1-\beta} \rho_M(z_1, \ldots, z_M) P\left(\frac{z_{M+1} - \beta z_M}{1-\beta} \right). \qquad (19)
\end{aligned}$$

Successively using this relation and taking $\rho_1(z) = \sigma(z)$ into account, we obtain the expression for the function $\rho_M(z_1, \ldots, z_M)$:

$$\rho_M(z_1, z_2, \ldots, z_M) = \frac{1}{[1-\beta]^{M-1}} \overbrace{P\left(\frac{z_M - \beta z_{M-1}}{1-\beta} \right) P\left(\frac{z_{M-1} - \beta z_{M-2}}{1-\beta} \right) \cdots P\left(\frac{z_2 - \beta z_1}{1-\beta} \right)}^{M-1} \sigma(z_1). \qquad (20)$$

In the case under consideration, $\sigma(z) = P(z)$, where $P(z)$ is defined by formula (2). Assuming that $M = N$, we obtain the final expression for the complete joint distribution function of random process (11):

$$\rho(z_1, z_2, \ldots, z_N) = \overbrace{P\left(\frac{z_N - \beta z_{N-1}}{1-\beta} \right) P\left(\frac{z_{N-1} - \beta z_{N-2}}{1-\beta} \right) \cdots P\left(\frac{z_2 - \beta z_1}{1-\beta} \right)}^{N-1} \frac{P(z_1)}{[1-\beta]^{N-1}}, \qquad (21)$$

$$P(z) = \frac{1}{\pi} \frac{\Delta}{\Delta^2 + z^2}.$$

4. Correlated Lloyd model

We now calculate the average Green's function of the disordered chain with Hamiltonian (1), where the site energies are the implementation of correlated random process (11). As in Sec. 2, without loss of generality, we consider an arbitrary diagram D passing through sites whose numbers can be assumed to be arranged in ascending order: $1 \le n_1 < n_2 < \cdots < n_q \le N$. The average value $\langle f_D \rangle$ of factor (6) of the considered diagram is now defined by a formula differing from (8):

$$\begin{aligned}
\langle f_D \rangle = \int dz_1\, dz_2 \cdots dz_N \left(\frac{1}{E + i\delta - z_{n_1}} \right)^{g_1} \times \\
\times \left(\frac{1}{E + i\delta - z_{n_2}} \right)^{g_2} \cdots \left(\frac{1}{E + i\delta - z_{n_q}} \right)^{g_q} \rho(z_1, z_2, \ldots, z_N), \qquad (22)
\end{aligned}$$

where $\delta \to +0$ and the function $\rho(z_1, z_2, \ldots, z_N)$ for the considered correlated Lloyd model has form (21). Because $\delta > 0$, the factors in brackets in relation (22) represented as the functions of complex $z_{n_1}, \ldots, z_{n_q}$, *have no singularities in the lower half-plane of complex* $z_{n_1}, \ldots, z_{n_q}$. This allows using formula (7) to calculate the integrals in (22) as follows.

We integrate in relation (22) over all z_j, $n_q < j \le N$. Then, the first $N-n_q$ factors of the type $P((z_N - \beta z_{N-1})/(1-\beta))$, $P((z_{N-1} - \beta z_{N-2})/(1-\beta)), \ldots$ of function (21) in this relation vanish, and the denominator $[1-\beta]^{N-1}$ is replaced with $[1-\beta]^{n_q-1}$. Integrating over z_{n_q} involves only the Lorentzian $P((z_{n_q} - \beta z_{n_q-1})/(1-\beta))$ contained in (21) and can be done using formula (7). If this formula is used, then we can verify that the indicated integration corresponds to multiplying by the factor $1-\beta$ and to replacing $z_{n_q} \to \beta z_{n_q-1} - i\Delta(1-\beta)$ in the factor $(1/(E + i\delta - z_{n_q}))^{g_q}$ in the integrand in (22). In this case, the pole of this function (with respect to the argument z_{n_q-1}) is located in the upper half-plane as before, which allows the next similar integration over z_{n_q-1}. The factor $P((z_{n_q-1} - \beta z_{n_q-2})/(1-\beta))$ depends on this variable in the function of joint probability density (21). As in the preceding case, integration over z_{n_q-1} corresponds to multiplying by the factor $1-\beta$ and to replacing $z_{n_q-1} \to \beta z_{n_q-2} - i\Delta(1-\beta)$ in the factor $(1/(E + i\delta - \beta z_{n_q-1} + i\Delta(1-\beta)))^{g_q}$ arising as a result of the preceding integration, and so on. Consequently, each new integration over z_j with a lower number j corresponds to multiplying by $1-\beta$ (i.e., to the cancellation of such a factor in the denominator of expression (21)) and to replacing $z_j \to \beta z_{j-1} - i\Delta(1-\beta)$ of the argument in the last factor. When the number i of the integration variable decreases such that $j = n_{q-1}$, further integrations can be performed similarly (i.e., simply replacing arguments), only the replacements indicated above must now also be done in the factor $(1/(E + i\delta - z_{n_{q-1}}))^{g_{q-1}}$.

We therefore conclude that the integration over all variables in (22) corresponds to successively replacing the symbols $z_{n_q}, \ldots, z_{n_2}, z_{n_1}$ in (22) in accordance with the rules given above. The indicated replacements for the symbol z_{n_q} are done as follows:[2]

$$
\begin{aligned}
&z_{n_q} = \beta z_{n_q-1} - i\Delta(1-\beta), \qquad \text{where}\\
&z_{n_q-1} = \beta z_{n_q-2} - i\Delta(1-\beta), \quad \text{where}\\
&z_{n_q-2} = \beta z_{n_q-3} - i\Delta(1-\beta), \quad \text{where}\\
&\quad\vdots\\
&z_3 = \beta z_2 - i\Delta(1-\beta), \qquad \text{where}\\
&z_2 = \beta z_1 - i\Delta(1-\beta).
\end{aligned}
\tag{23}
$$

The last integration over z_1 corresponds to replacing $z_1 \to -i\Delta$ because this integration is done with the function $P(z_1)$ (see (21)). It is easy to see that chain of replacements (23) is simplified for $z_1 = -i\Delta$ and corresponds to replacing $z_{n_j} = -i\Delta$, $j = 1, 2, \ldots, n_q$. Average (22) hence simply corresponds to replacing all the quantities z_{n_j}, $j = 1, 2, \ldots, q$, with $-i\Delta$, as in the case of uncorrelated Lloyd model (8), and we conclude that *the averaged Green's function of the correlated Lloyd model with site energies of form* (11) *is independent of the correlation radius* $R = -1/\log\beta$ *and turns out to be the same as in the no-correlation case*, i.e., is defined by formula (9).

The fact that there is no dependence of the average Green's function on the correlation radius $R = -1/\log\beta$ shows the peculiar scale invariance of the considered correlated Lloyd model because the spatial

[2]Because n_q is the greatest of the numbers n_j of the sites of the considered diagram, the replacements of the other symbols are also contained in this sequence.

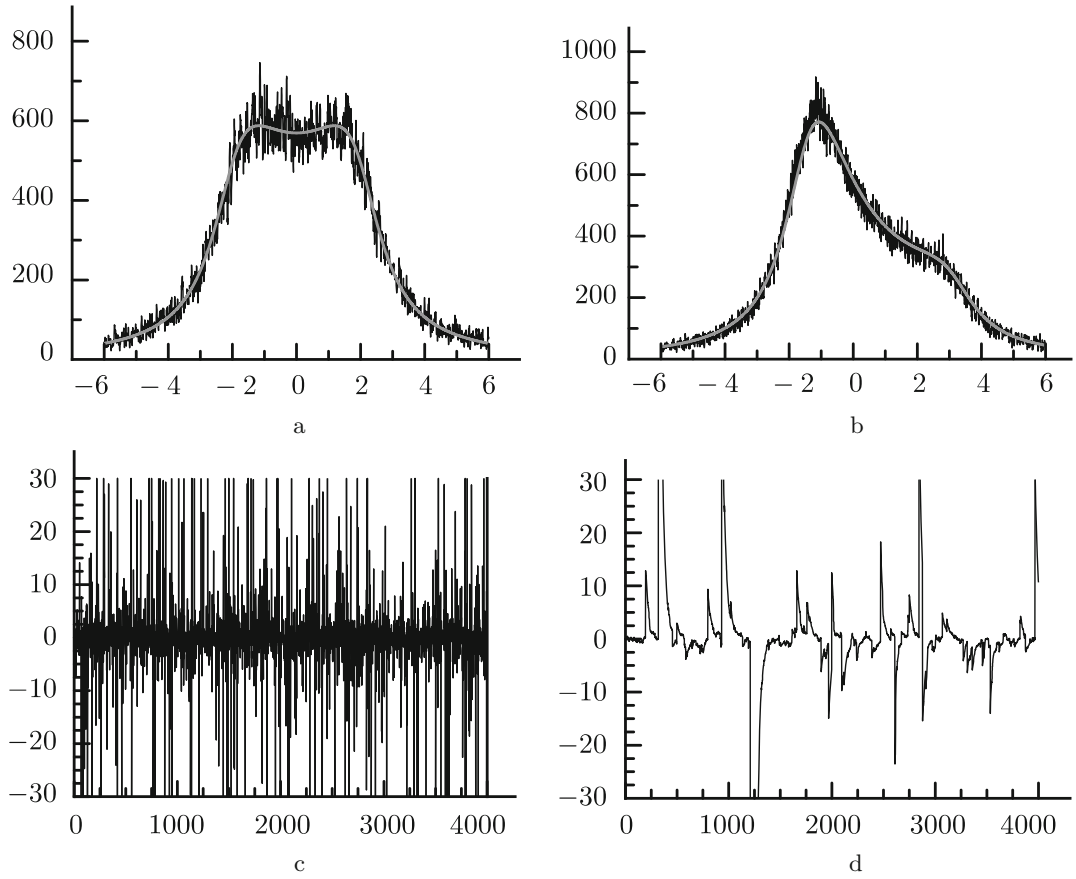


Fig. 2. Correlated Lloyd model for different correlation radii R and different forms of the function $w(r)$ describing nondiagonal Hamiltonian elements (1). The density of states of Hamiltonian (1) for (a) $w(r) = \delta_{1,r} + \delta_{-1,r}$ and (b) $w(r) = v_0 e^{-|r/R_1|}$ ($R_1 = 1$, $v_0 = e^{1/R_1}$): the noise dependences were obtained by numerical diagonalization of Hamiltonian (1), the smooth curves were calculated using the respective formulas (24) and (26), and the densities of states were obtained for the correlation radii (a) $R = 0.1$ and (b) $R = 30$. The realization of random potentials for the correlation radii (c) $R = 0.1$ and (d) $R = 30$: the abscissa represents the site number n, the ordinate represents ε_n in (11), $\Delta = 1$ in all the cases, and the size of the random matrices in the numerical calculations is $N = 4000$.

dependence (the dependence on the site number) of the random potential ε_n given by (11) turns out to be significantly different for different R (see Figs. 2c and 2d).

We illustrate the obtained result by examples in which the Green's function $\mathbf{G}^{\mathrm{od}}(\Omega)$ given by (10) can be calculated analytically. The first (well-known) example of such kind is the case where the Hamiltonian has the form of the banded matrix $H^{\mathrm{tb}}_{rr'} = \delta_{r,r'+1} + \delta_{r,r'-1}$ and the diagonal elements of the Green's function are $G^{\mathrm{od}}_{nn}(\Omega) = [\Omega^2 - 4]^{-1/2}$. In accordance with the obtained results, the average density of states $\rho^{\mathrm{tb}}_{\Delta}(E) = -\pi^{-1} \operatorname{Im} \operatorname{Sp}\langle \mathbf{G} \rangle$ of random matrix (1) with the site energies ε_r of form (11) and for $w(r) = \delta_{r,1} + \delta_{r,-1}$ is independent of the correlation radius $R = -1/\log \beta$ and equal to

$$\rho^{\mathrm{tb}}_{\Delta}(E) = -\frac{N}{\pi} \operatorname{Im} \frac{1}{\sqrt{(E + i\Delta)^2 - 4}}. \tag{24}$$

The second (less-known) example is the case where the matrix of the ordered Hamiltonian has the form

$H^{\mathrm{ex}}_{r,r'} = v_0 e^{-|r-r'|/R_0}$. In this case, the matrix of the Green's function becomes[3]

$$\Gamma_{r,r'}(\Omega) = Ae^{-|r-r'|\eta} + \frac{\delta_{r,r'}}{\Omega}, \tag{25}$$

where

$$A \equiv \frac{V}{(\Omega - V)\Omega\sqrt{1-T^2}}, \qquad V \equiv v_0 \tanh\frac{1}{R_0}, \qquad \frac{1}{T} \equiv \frac{V-\Omega}{\Omega}\cosh\frac{1}{R_0}, \qquad \cosh\eta \equiv \left|\frac{1}{T}\right|,$$

and the density of states of random matrix (1) for $w(r) = v_0 e^{-|r-r'|/R_0}$ and ε_r given by (11) is determined by the expression[4]

$$\rho^{\mathrm{ex}}_{\Delta}(E) = -\frac{N}{\pi}\operatorname{Im}\Gamma_{00}(E + i\Delta + v_0). \tag{26}$$

In these two cases, the numerical diagonalization of random matrices of form (1) for different correlation radii R showed that the density of states is described by formulas (24) and (26) (up to noise) and is really independent of R, although the form of random potential (11) varies substantially (Fig. 2).

5. Conclusions

We have presented an exact calculation of the average Green's function for the correlated Lloyd model. We showed that for a random potential of the considered form, the average Green's function is independent of the parameter of the random potential playing the role of the correlation radius of the last. The obtained result was illustrated with numerical calculations.

Acknowledgments. The work was supported by the Russian Federation Ministry of Education and Science (Contract No. 11.G34.31.0067 with St. Petersburg State University and supervisor of studies A. V. Kavokin) and St. Petersburg State University (Research Grant No. 11.38.67.2012).

REFERENCES

1. F. M. Izrailev, A. A. Krokhin, and N. M. Makarov, *Phys. Rep.*, **512**, 125–254 (2012); arXiv:1110.1762v1 [cond-mat.dis-nn] (2011).
2. M. Titov and H. Schomerus, *Phys. Rev. Lett.*, **95**, 126602 (2005).
3. L. I. Deych, M. V. Erementchouk, and A. A. Lisyansky, *Phys. B*, **338**, 79–81 (2003).
4. A. Croy, P. Cain, and M. Schreiber, *Eur. Phys. J. B*, **82**, 107–112 (2011).

[3]Formula (25) is not very popular, and we have been unable to find a reference to it. Therefore, we briefly describe how it is obtained. For the chain closed to form a circle, the function $\mathbf{\Gamma}(u)$ only depends on the difference between its subscripts: $\Gamma_{l,l'}(u) = \Gamma_r(u)$, $r \equiv l - l'$. Standard manipulations lead to an expression of form (10) for $\Gamma_r(u)$:

$$\Gamma_r(u) = \frac{1}{N}\sum_{q=-\pi}^{\pi}\frac{e^{-iqr}}{u - J_q},$$

where $J_q \equiv \sum_r w(r)e^{iqr}$. In the case of the exponential function $w(r)$, the function J_q is the sum of the geometric progression and is calculated exactly. In the given formula for $\Gamma_r(u)$, the summation is over the first Brillouin zone with the step $\Delta q = 2\pi/N$ and can be replaced with integration in the thermodynamic limit $N \to \infty$. If the integrand periodicity is used, then the integration can be extended to the $2M{+}1$ Brillouin zone, and the sought integral can be obtained by dividing the result by $2M + 1$:

$$\frac{1}{N}\sum_{q=-\pi}^{\pi}\cdots \to \frac{1}{2\pi}\int_{-\pi}^{\pi} dq \cdots = \frac{1}{2\pi(2M+1)}\int_{-\pi(1+2M)}^{\pi(2M+1)} dq \ldots.$$

For the exponential function $w(r)$, the function J_q is such that the last integral can be calculated as $M \to \infty$ using residues by means of the standard closure of the contour along a large circle. In this case, the number of residues increases in proportion to M, which provides the existence of the limit as $M \to \infty$.

[4]The appearance of the shift v_0 in this formula is related to the fact that Green's function (25) was obtained for the matrix $\mathbf{H}^{\mathrm{ex}}$, whose diagonal elements are nonzero and equal to v_0.

5. O. Derzhko and J. Richter, *Phys. Rev. B*, **59**, 100–103 (1999).
6. O. Derzhko and J. Richter, *Phys. Rev. B*, **55**, 14298–14310 (1997).
7. V. A. Malyshev, A. Rodriguez, and F. Dominguez-Adame, *Phys. Rev. B*, **60**, 14140–14146 (1999).
8. F. A. B. F. de Moura and M. L. Lyra, *Phys. Rev. Lett.*, **81**, 3735–3738 (1998).
9. D. H. Danlap, H.-L. Wu, and P. W. Phillips, *Phys. Rev. Lett.*, **65**, 88–91 (1990).
10. G. G. Kozlov, *Theor. Math. Phys.*, **171**, 531–540 (2012).
11. G. G. Kozlov, *Appl. Math.*, **2**, 965–974 (2011).
12. F. J. Dyson, *Phys. Rev.*, **92**, 1331–1338 (1953).
13. P. Lloyd, *J. Phys. C*, **2**, 1717–1725 (1969).
14. C. R. Gochanour, H. C. Andersen, and M. D. Fayer, *J. Chem. Phys.*, **70**, 4254–4271 (1979).
15. G. G. Kozlov, "The watching operators method in the theory of Frenkel exciton: Novel criterion of localization and its exact calculation for the non diagonal disordered 1D chain's zero-state," arXiv:cond-mat/9909335v1 (1999).
16. I. M. Lifshits, S. A. Gredeskul, and L. A. Pastur, *Introduction to the Theory of Disordered Systems* [in Russian], Nauka, Moscow (1982); English transl., Wiley-Interscience, New York (1988).
17. B. M. Miller and A. R. Pankov, *Theory of Stochastic Processes in Examples and Problems* [in Russian], Fizmatlit, Moscow (2007).
18. E. S. Ventsel' and L. A. Ovcharov, *Theory of Random Processes and Its Engineering Application* [in Russian], Vyshaya Shkola, Moscow (2000).