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Superconductivity in semiconductor structures: The excitonic mechanism

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ABSTRACT

We theoretically study the dependency of the superconductivity transition critical temperature (T_C) on the electron and exciton–polariton densities in layered systems, where superconductivity is mediated by a Bose-Einstein condensate of exciton–polaritons. The critical temperature increases with the polariton density, but decreases with the electron gas density, surprisingly. This makes doped semiconductor structures with shallow Fermi energies better adapted for observation of the exciton–polariton-induced superconductivity than metallic layers. For realistic GaAs-based microcavities containing doped and neutral quantum wells we estimate T_C as close to 50 K. Superconductivity is suppressed by magnetic fields of the order of 4 T due to the Fermi surface renormalization.

1. Introduction

High temperature superconductivity (HTSC) has been desperately searched for during decades since the appearance of the seminal work of Bardeen–Cooper–Schrieffer (BCS) [1] in early 50 s. Among many different paths physicist have tried to achieve it, the excitonic mechanism of superconductivity (SC) deserves particular attention [2–4]. According to Ginzburg [5,6], the excitons are expected to be suitable for realization of HTSC. The reason is that the characteristic energy, above which the electron attraction mediated by excitons vanishes, is several orders of magnitude higher than the Debye energy limiting the attraction mediated by phonons.

Despite optimistic expectations, to the best of our knowledge, the exciton mechanism of SC has never worked until now, most likely due to the reduced retardation effect [4,7]. Phonons in the BCS model are very slow compared to electrons on the Fermi surface. Hence there is a strong retardation effect in the phonon-mediated electron–electron attraction, so that the size of a Cooper pair is very large (of the order of 100 nm), and the Coulomb repulsion may be neglected at such distances. In contrast, excitons with wave-vectors comparable with the Fermi wave-vector in metals are very fast quasi-particles. Therefore the replacement of phonons with excitons leads to the loss of retardation and smaller sizes of Cooper pairs, that

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is why the Coulomb repulsion starts playing an important role. In realistic multilayer structures the Coulomb repulsion appears to be stronger than the exciton-mediated attraction so that Cooper pairs cannot be formed. In literature [8,9] one finds reports on layered metal-insulator structures where SC occurs at 50 K in the layered metal-insulator structures, nevertheless there is still no evidence that the excitonic mechanism is responsible for this effect. Recently, the novel mechanism to achieve superconductivity mediated by exciton–polaritons has been proposed in Refs. [10,11]. Exciton–polaritons are quasi-particles that arise due to the strong coupling of excitons with light. Particularly interesting phenomena related to exciton–polaritons have been observed in semiconductor quantum wells (QW) embedded in microcavity [12,13]. Bose–Einstein condensation of cavity polaritons at room temperature has been observed [14–19], making the exciton–polariton a promising boson to bind Cooper pairs at high temperatures. Moreover, it has been shown that the strength of electron–electron interactions mediated by a condensate of cavity polaritons can be controlled optically.

The systems considered previously in Refs. [10,11] consist of microcavities where free electrons in a thin layer interact with exciton–polaritons contained in the adjacent semiconductor layer. Both layers are brought sufficiently close to each other to assure the efficient coupling between the electrons and exciton–polaritons. In this way, phonons are replaced with the excitations of an exciton–polariton condensate providing the exciton-mediated attraction of free electrons. While the retardation effect characteristic of the weak-coupling BCS model is essentially suppressed in this regime, the exciton-mediated attraction appears to be strong enough to overcome the Coulomb repulsion for Cooper pairs of a characteristic size of 10 nm. In comparison to the mechanism considered by Bardeen [1] and Ginzburg [5,6], electron–electron attraction mediated by excitons is much stronger in the presence of the exciton–polariton bosonic condensate for two reasons: firstly, the exchange energy needed for creation of an excited state of the condensate is much smaller than the energy needed to create a virtual exciton. Secondly, the exciton–electron interaction strength increases proportionally to the occupation number of the condensate. This exciton–polariton mechanism of superconductivity was studied theoretically in a model structure where the electron–electron attraction potential was calculated and then substituted into the gap equation that yielded the critical temperature of the superconductivity phase transition. The proof of concept calculation showed a high potentiality of the excitonic mechanism of SC.

In order to proceed with the experimental verifications of this prediction, several issues still need to be clarified. Namely, it has been unclear how the electron density in the two-dimensional electron gas (2DEG) QW influences the critical temperature T_C and what structure is the most appropriate for experimental observation of the predicted effect: the one where the metallic layer is put in contact with the semiconductor, or an entirely semiconductor multilayer structure containing doped and undoped QWs.

2. Results

In this manuscript we analyze the behavior of superconducting gap and T_C as a function of exciton–polariton and electron densities and conclude on the most convenient structure design for observation of the exciton-mediated SC. The system we study is a microcavity where an electron gas confined to a quantum well (2DEG) interacts with a polariton condensate localized in an adjacent semiconductor QW, as shown in Fig.1. The microscopic Hamiltonian that describes this system is derived in Ref. [11]. Here we only need the expression for the reduced Hamiltonian that appears after the Bogoliubov transformation and describes the coupling of electrons via bogolons, excitations of a polariton condensate:

$$H = \sum_{\mathbf{k}} E_{el}(\mathbf{k})\sigma^{\dagger}_{\mathbf{k}}\sigma_{\mathbf{k}} + \sum_{\mathbf{k}} E_{bog}(\mathbf{k}) b^{\dagger}_{\mathbf{k}} b_{\mathbf{k}} + H_c + \sum_{\mathbf{k},\mathbf{q}} M(\mathbf{q})\sigma^{\dagger}_{\mathbf{k}}\sigma_{\mathbf{k}+\mathbf{q}}\left(b^{\dagger}_{-\mathbf{q}} + b_{\mathbf{q}}\right). \tag{1}$$

Here E_{el} is the free electron energy, the bogolon dispersion is given by the formula:

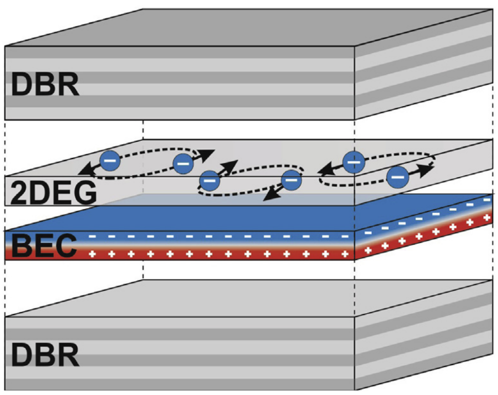

Fig. 1. The scheme of the model microcavity structure with an n-doped QW interacting with an exciton–polariton BEC localized in an adjacent QW.

$$E_{bog}(\mathbf{k}) = \sqrt{\widetilde{E}_{pol}(\mathbf{k})(\widetilde{E}_{pol}(\mathbf{k}) + 2UN_0A)}, \tag{2}$$

where $\widetilde{E}_{pol} = E_{pol}(\mathbf{k}) - E_{pol}(0)$, U is the polariton–polariton interaction potential, N_0 is the concentration of exciton–polaritons in the condensate, A is a normalization area, H_c is the Coulomb repulsion term, $\mathbf{q} = \mathbf{k}_1 - \mathbf{k}_2$, where $\mathbf{k}_1$ and $\mathbf{k}_2$ are the momenta of two interacting electrons at the Fermi surface, $q = \sqrt{2k_F(1+\cos\theta)}$.

The renormalized bogolon–electron interaction in Eq. (1) is given by $M(\mathbf{q})$. It is important that $M(\mathbf{q}) \sim \sqrt{N_0}$. The exciton concentration can be controlled by the external optical pumping, which is why the strength of Cooper coupling in exciton-mediated superconductors may be tuned in large limits.

The total effective interaction potential including Coulomb repulsion is:

$$V_{eff}(\omega) = \frac{A\mathscr{N}}{2\pi}\int_0^{2\pi}[V_A(\mathbf{q},\omega) + V_C(\mathbf{q})]\mathrm{d}\theta, \tag{3}$$

where $\mathscr{N} = m_e/\pi\hbar^2$, $V_A(\mathbf{q},\omega)$ is the effective electron interaction given in Refs. [10,11], the Coulomb repulsion is given by $V_C(q) = e^2/2\varepsilon A(|\mathbf{q}| + \kappa)$, κ is the screening constant.

In Refs. [10,11] it is shown that the magnitude of the attraction potential increases linearly with N_0. This is illustrated by Fig. 2a, where it is clear that the higher N_0 is, the higher the magnitude is and the larger the attraction region is. On the contrary, Fig. 2b shows that a high electron density leads to the decreasing magnitude of the negative part of potential that corresponds to the attraction between electrons. This effect is observed in a wide range of polariton density values. The only essential limitation to this mechanism of SC is the Mott transition from an exciton (exciton–polariton) condensate to an electron–hole plasma. The obtained result is significant for designing microstructures exhibiting HTSC, as it demonstrates advantage of fully semiconductor heterostructures over metal-semiconductor ones.

To obtain the critical temperature of the SC phase transition one needs to substitute this potential into the gap equation:

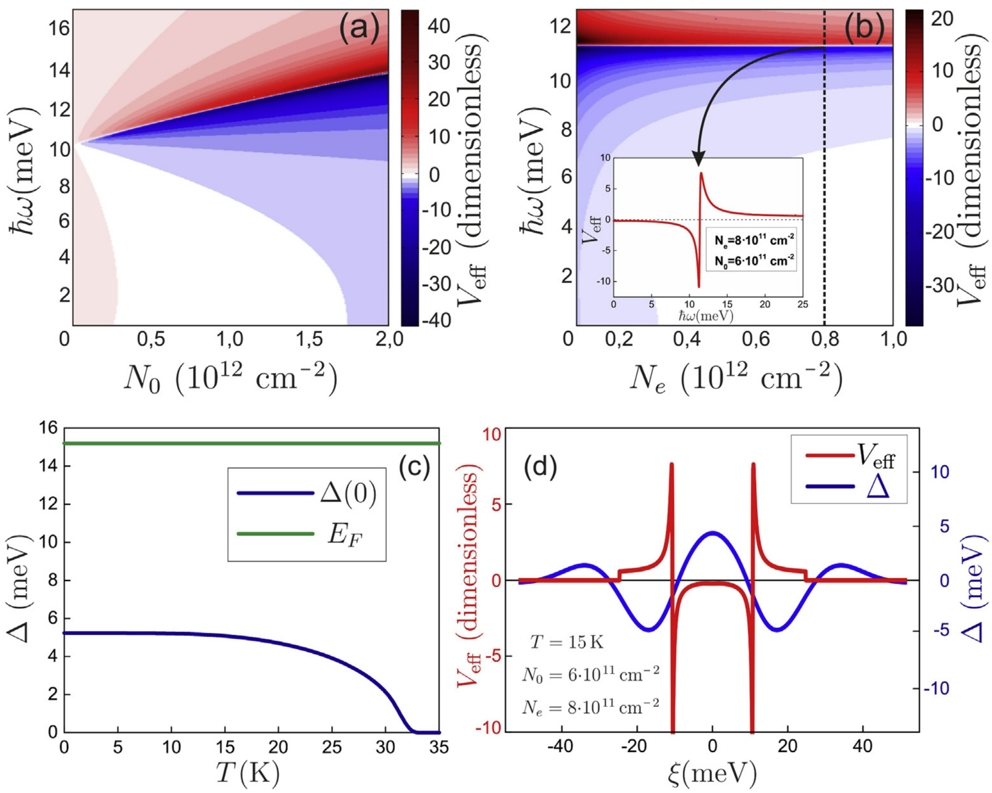

Fig. 2. The magnitude of effective interaction potential as a function of a) polariton density $\mathbf{N_0}$ and b) electron density $\boldsymbol{N_e}$ in 2DEG quantum well. The color shows the magnitude in dimensionless units. Blue region corresponds to the effective attraction between electrons, red region represents the repulsion. The inset presents the profile of the potential at the particular density $\boldsymbol{N_e}$. Graphs (c) and (d) show the solution of the gap-equation. (c) $\Delta(\mathbf{0})$ as a function of temperature. The critical temperature $\boldsymbol{T_C}$ in this case is equal to 33 K. (d) solution of the Eq. (5) at $T = 15$ K. The results are presented for the potential with $N_e = 8\times10^{11}$ cm^{-2} and $N_0 = 6\times10^{11}$cm^{-2}. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$$\Delta(\xi, T) = \int_{-\infty}^{\infty} \frac{U_0(\xi - \xi\prime)\Delta(\xi', T)\tanh(E/2k_BT)}{2E} d\xi', \tag{4}$$

where $E = \sqrt{\Delta(\xi\prime, T)^2 + \xi'^2}$ and U_0 is the electron–electron interaction potential.

In the case of a strongly non-monotonous potential $U_0 = V_{eff}(\omega)$ shown in Fig. 2a and b this equation may be solved numerically. Here we solve it using the iteration method. The example of solution is shown in Fig. 2c and d.

We assume that only the electrons on the Fermi surface form Cooper pairs. Here it means that only the point $\Delta(0)$ has a physical meaning. If $\Delta(0) > 0$, Cooper pair can be formed. The critical temperature T_C can be defined as the temperature below which $\Delta(0)$ is non-zero. Fig. 3 represents the critical temperature of the SC transition as a function of electron concentration in a 2DEG QW. The green line shows the temperature that corresponds to the Fermi-energy, the other lines represent the dependencies of T_C on the electron density for different values of exciton–polariton density. One can see that increasing N_e leads to the reduction of the critical temperature. The colored area shows the range of parameters where our theory is applicable. We note that our model has two important limitations. Firstly, the thermal energy of electrons at the critical temperature must be lower than the Fermi energy. Otherwise, one cannot assume that electrons forming the Cooper pairs are located at the Fermi surface. Also, the absolute value of the gap-energy must be lower than the Fermi energy. In Fig. 3 the area of validity of our approach is limited by $E_F = k_BT$ line. The density $N_0 = 4 \times 10^{12}$ cm^{-2} is apparently beyond the Mott transition threshold, therefore high T_C predicted by this line is unrealistic and is presented only for showing the tendency of T_C growth. On the other hand, the exciton–polariton density $N_0 = 4 \times 10^{11}$ cm^{-2} is achievable in realistic QW structures based on GaAs, so critical temperatures of the order of a few tens of Kelvin must be achievable in semiconductor structures.

The superconducting currents may be observed in our structures until the critical current density is achieved. It can be conveniently derived from the superconducting gap $\Delta(0)$ as [20].

$$j_c = \frac{eN_e\Delta(0)}{\hbar k_F}, \tag{5}$$

Fig.4a shows j_c calculated as a function of the electronic density N_e and temperature. One can see that the highest current density appears at the lowest concentrations and lowest temperatures on the graph, that fully agrees with calculations.

Let us now discuss the behavior of exciton-mediated superconductors in the presence of external magnetic fields. It is known that in bulk superconductors the Meissner effect exists until the critical magnetic field is achieved. This field is linked to the critical current. Namely, the critical field induces a surface current equal to j_c. Once the surface currents induced to provide a full screening of the magnetic field inside the superconductor exceed the critical current, the superconductivity is suppressed. In 2D system we consider the magnetic field direction perpendicular to the QW plane. The superconducting layer is much thinner than the typical penetration length of the magnetic field into the superconductor. So Meissner effect can't be observed. The superconductivity is still suppressed by the magnetic field in this case, but the gap vanishes at the critical field B_{cr} [21]. In order to find B_{cr} and the critical temperature we will account for the magnetic field in the gap equation, using the condition: $\Delta(0, T, B_{cr}) = 0$.

The field reduces the density of electronic states in the 2DEG layer which leads to the increase of the radius of the Fermi circle. A minor effect is the modification of electron–exciton interaction potential due to the shrinkage of the exciton Bohr radius. To account for the magnetic field effect on k_f, we use the expression for the radii of the circles in the reciprocal space, that correspond to Landau levels in the quasi-classical approximation [22]:

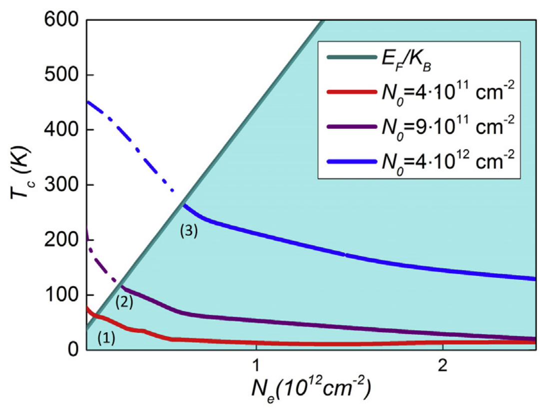

Fig. 3. The dependency of $\mathbf{T_C}$ on the electron density in 2DEG QW, plotted for three different polariton densities $\mathbf{N_0}$. Dashed parts of the curves show the region where the theory is not applicable. Red curves (1,2) represent the parameters of a condensate that are achievable in a realistic GaAs-based semiconductor structures. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

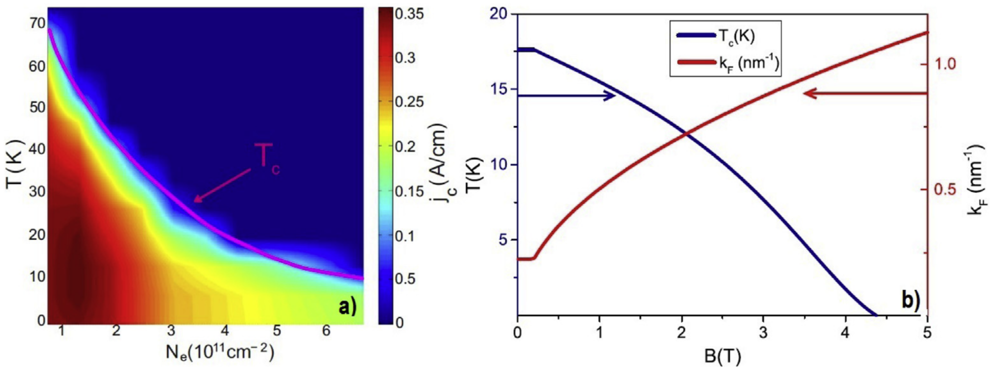

Fig. 4. (a) The dependency of the critical current $\mathbf{j_c}$ on the temperature and electron concentration. (b) Fermi wave-vector (red curve) and critical temperature (blue curve) as a function of magnetic field B. $N_e = 8 \times 10^{11}$ cm^{-2}, $N_0 = 6 \times 10^{11}$ cm^{-2}. The Dingle broadening of Landau Levels Γ is taken to be 0.3 meV, that corresponds to the cyclotron energy $\hbar\omega_c$ at $B = 0.2$ T. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$$k_p^2 = \left(p + \frac{1}{2}\right)\frac{2eB}{\hbar c}, p = 0, 1, 2... \tag{6}$$

Electrons may occupy quantum states in the Γ vicinity of these circles, where Γ is the Dingle broadening of Landau levels dependent on the structural disorder and scattering processes. The area occupied by electrons in the reciprocal space at each circle at zero temperature may be found as

$$S_p = 2\pi k_p \delta k_p, \tag{7}$$

where $\delta k_p = \frac{2m\Gamma}{\hbar^2 k_p}$. The Fermi wave-vector is expressed as $k_F = k_M$, where the index M can be found from the condition:

$$\frac{2}{(2\pi)^2}\sum_{p=0}^{M-1} S_p < N_e \le \frac{2}{(2\pi)^2}\sum_{p=0}^{M} S_p. \tag{8}$$

Fig. 4b shows k_F and T_c as functions of magnetic field B for the fixed electron and polariton densities. All parameters are the same that we used for potential calculation for GaAs-structure. In this case $N_e = 8 \times 10^{11}$ cm^{-2}, $N_0 = 6 \times 10^{11}$ cm^{-2}, the Dingle broadening of Landau Levels is taken to be to 0.3 meV. At low magnetic fields given by the condition $\hbar\omega_c < \Gamma$ we assume that $k_F = k_F(B = 0)$, neglecting the weak oscillations of k_F due to the oscillating electron density of states [23].

3. Discussion

Contrary to the previous expectations, fully semiconductor structures, combining doped and undoped quantum wells provide higher critical temperatures than metal-semiconductor structures. This can be explained by the fact that exciton-mediated attraction weakens with the increasing of the Fermi energy faster than the Coulomb repulsion does. In the absence of magnetic field we predict the critical temperatures of the order of 50 K in realistic GaAs-based microcavities.

We show that magnetic fields strongly increase the Fermi wave-vector k_F, which is why the critical temperature decreases and eventually vanishes at $B_c \approx 4$ T. The increase of k_F accounts for the reduction of the effective area occupied by each electron in the real space due to the cyclotron motion. We note, that the validity of the quasi-classical approximation is limited at strong quantizing magnetic fields. As long as quantum Hall regime is not established and the number of filled Landau levels $N \gg 1$, quasi-classical approach is applicable. In our case, the number of occupied Landau levels is over 10 even at $B \approx 4$ T, which allows one to consider the quasi-classical result as a trustworthy approximation. Other effects which may influence B_c include the electron Zeeman splitting and edge current effects. GaAs/AlGaAs quantum wells are characterized by low Lande factors $g \approx 0.01$ depending on the actual heterostructure parameters. The electron Zeeman splitting in the considered range of magnetic fields $B \approx 4$ T is of the order of a few μeV and is negligible with respect to other characteristic energy scales. Edge current effects are beyond the scope of the present work. In conclusion, multilayer semiconductor heterostructures appear to be promising candidates for the observation of exciton-mediated superconductivity.

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