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Light scattering in a medium with fluctuating gyrotropy: Application to spin-noise spectroscopy

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The spin-noise signal in the Faraday-rotation-based detection technique can be considered equally correctly either as a manifestation of the spin-flip Raman effect or as a result of light scattering in the medium with fluctuating gyrotropy. In this paper, we present rigorous description of the signal formation process upon heterodyning of the field scattered due to fluctuating gyrotropy. Along with conventional single-beam experimental arrangement, we consider here a more complicated, but more informative, two-beam configuration that implies the use of an auxiliary light beam passing through the same scattering volume and delivering additional scattered light to the detector. We show that the signal in the spin-noise spectroscopy (SNS) arising due to heterodyning of the scattered field is formed only by the scattered field components the wave vectors of which coincide with those of the probe field. Therefore, in principle, the detected signal in SNS can be increased by increasing overlap of the two fields in the momentum space. We also show that, in the two-beam geometry of SNS, contribution of the auxiliary (tilted) beam to the detected signal is produced only by the region of overlap between the two beams in real space and can be expressed analytically as the Fourier transform of the spatial correlation function of the gyrotropy at the difference of their wave vectors. These features of the two-beam geometry can be used for tomographic measurements in spin systems (a more sophisticated version of three-dimensional tomography proposed earlier) and for studying spatial spin correlations by means of noise spectroscopy.

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I. INTRODUCTION

Spin-noise spectroscopy (SNS), first realized in [1], has turned nowadays into a powerful method of studying magnetic resonance and spin dynamics in atomic and semiconductor systems (see, e.g., [2–5]). The most fascinating results of application of the SNS with the greatest progress in sensitivity of the measurements were achieved in physics of semiconductor structures, where the novel technique has allowed one not only to considerably move ahead in the magnetic resonance spectroscopy but also to discover fundamentally new opportunities of research. Specifically, it has been established that optical spectroscopy of spin noise (that implies measuring wavelength dependence of the spin-noise power) makes it possible to decipher the inner structure of optical transitions [6]. Correlation nature of the SNS allowed one to realize, on its basis, a sort of pump-probe spectroscopy [7]. Effective dependence of the spin-noise signal on the light-power density (on the beam cross section) was used to demonstrate SNS-based three-dimensional (3D) tomography [8,9]. Due to high sensitivity of the SNS, it appeared possible to detect magnetic resonance of quasifree carriers in a single quantum well, 20 nm thick [10], to observe the spin-noise spectrum of a single hole spin in a quantum dot [11], and to realize magnetometry of local magnetic fields (including the field of polarized nuclei) in a semiconductor [12,13]. Due to these remarkable capabilities of the new technique, it acquired a great popularity during the last decade.

At the same time, certain fundamental aspects of the spin-noise-based magnetic resonance are not so far fully understood. In the first publication [1], the polarization signal detected with the aid of a balanced circuit was ascribed entirely to fluctuations of the polarization plane azimuth of the probe beam, with no reference to the optical heterodyning process. It is interesting to note, in this connection, that practically the same measuring system was considered earlier [14] as a balanced optical heterodyne detector with orthogonal polarizations of the local oscillator's and signal's waves.

In 1983 [15], it was shown theoretically that the effect of magnetic resonance in the Faraday rotation noise is closely connected with the spin-flip Raman scattering, and the detected signal is the result of heterodyning of the light scattered in the forward direction, with the local oscillator provided by the probe field. In the framework of this model, the standard experimental geometry of SNS, which implies collecting only the scattered light lying within the solid angle of the probe beam, may appear to be far from optimal. In other words, it looks like the detected signal, in the SNS, can be considerably increased by collecting the scattered light more efficiently. First experiments carried out in this direction [16] and our preliminary analysis of the problem have shown that a favorable solution of this experimental task can be achieved only with allowance for all the factors affecting the heterodyning process (wave fronts of the reference and scattered waves, shape of the beam, volume of the scattering medium, shape and dimensions of the photosensitive surface, correlation properties of the gyrotropy, etc.). Actually, this problem, which we consider to be fundamental for the SNS method, is rather complicated and needs to be analyzed carefully and rigorously, with the results of the treatment applicable to real experimental conditions. In our opinion, computational details of such a treatment and particularities of the used approximations are also highly important.

In this paper, we present such a treatment for a focused Gaussian probe beam propagating through the medium with fluctuating gyrotropy and analyze in detail the mechanism of the intensity-noise signal formation due to heterodyning of the scattered field on the detector. We also propose a two-beam experimental arrangement, with the auxiliary light beam tilted with respect to the probe, that makes it possible to get information about the spatiotemporal correlation function of gyrotropy of the studied system (recall that in conventional SNS only the spatially averaged temporal correlation function is revealed).

The paper is organized as follows. In Sec. I, for completeness of the narrative, we present a brief explanation of what is the Gaussian beam and introduce a model of the polarimetric detector used in our further analysis. We show here that the detected signal in SNS is contributed only by the scattered field that, in the momentum space, coincides with that of the probe. In Sec. II, we present basics of the single-scattering theory, apply it to the medium with gyrotropy randomly modulated in space, and calculate the observed polarimetric signal. In Secs. III and IV, we calculate the noise signal observed in the two-beam configuration, when the auxiliary beam propagating though the medium at some angle to the main probe beam does not hit the detector and contributes to the signal only by its scattered field. We show that the spin-noise signal, under these conditions, is proportional to the Fourier component of the spatial correlation function of gyrotropy at spatial frequency equal to the difference between the two wave vectors. In Sec. V, we present calculations for the model of independent paramagnetic particles (spins) and show that the signal produced by the auxiliary tilted beam is of the same order of magnitude as the one produced by the main probe and, hence, can be easily detected using the same experimental setup.

II. DETECTING A POLARIMETRIC SIGNAL IN A GAUSSIAN BEAM

In the simplest version of the light-scattering problem, the probe beam can be taken in the form of a plane wave. However, in the SNS experiments under consideration, when two light beams are supposed to be used, with their spatial localization being of crucial importance, this approximation proves to be inappropriate. So, we will treat Gaussian beams the electric fields $\mathbf{E_p}(\mathbf{r})$ of which are defined by the expression

$$\mathbf{E_0}(\mathbf{r}) = e^{i[kz-\omega t]}kQ\sqrt{\frac{8W}{c}}\frac{(\cos\phi,\sin\phi,0)}{(2k+iQ^2z)} \times \exp\left[-\frac{kQ^2(x^2+y^2)}{2(2k+iQ^2z)}\right],\quad \mathbf{r}=(x,y,z) \quad (1)$$

where $\mathbf{r}\equiv(x,y,z)$, $k\equiv\omega/c$ (ω is the optical frequency and c is the speed of light), W is beam intensity, and the angle ϕ specifies beam polarization in the xy plane. Field (1) satisfies Maxwell's equations and represents the beam propagating along the z axis. The parameter Q defines the e-level half width $2w$ of the beam waist by the relationship $w=1/Q$ (w is assumed greater than the wavelength $\lambda=2\pi c/\omega$). In our estimations, we accept $\lambda\sim 1\ \mu$m and $w\sim 30\ \mu$m.

In the SNS experiments, we detect small fluctuations of the optical field polarization, and, therefore, to calculate correctly the SNS signal, we have to specify the model of the polarimetric detector. We suppose the detector to be composed of two photodiodes PD1 and PD2 (Fig. 1) arranged in two arms of the polarization beam splitter (BS). The output signal U is obtained by subtracting photocurrents of the two photodiodes and (to within some unimportant factors) is given by the

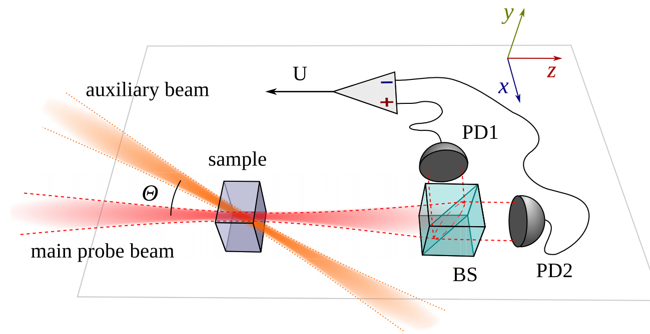

FIG. 1. The two-beam experimental arrangement. BS, polarization beamsplitter; PD1 and PD2, photodetectors.

expression

$$U=\frac{\omega}{2\pi}\int_0^{2\pi/\omega}dt\int_{-l_x}^{l_x}dx\int_{-l_y}^{l_y}dy[\mathrm{Re}^{\,2}E_x(x,y,L) - \mathrm{Re}^{\,2}E_y(x,y,L)], \quad (2)$$

where $E_{x,y}$ are the x and y components of the complex input optical field $\mathbf{E}$, $2l_{x,y}$ are the dimensions of sensitive areas of the photodiodes along the x and y directions. We ascribe physical sense to the real part of the complex optical field and, as seen from Eq. (2), the output signal U represents the difference between intensities of the input optical field in the x and y polarizations integrated over sensitive areas of the photodiodes and averaged over the optical period $2\pi/\omega$.

In our case, the input optical field $\mathbf{E}$ can be presented as a sum of the probe field $\mathbf{E_0}$ ($\mathrm{Re}\,\mathbf{E_0}\equiv\mathcal{E}_\mathbf{0}$) and the field $\mathbf{E_1}$ ($\mathrm{Re}\,\mathbf{E_1}\equiv\mathcal{E}_\mathbf{1}$) arising due to scattering of the probe beam by the sample with spatially fluctuating gyrotropy. Then, the first-order (with respect to $\mathbf{E_1}$) contribution u_1 to the polarimetric signal can be written as

$$u_1=\frac{\omega}{\pi}\int_0^{2\pi/\omega}dt\int_{-l_x}^{l_x}dx\int_{-l_y}^{l_y}dy[\mathcal{E}_{x0}(x,y,L)\mathcal{E}_{x1}(x,y,L) - \mathcal{E}_{y0}(x,y,L)\mathcal{E}_{y1}(x,y,L)]. \quad (3)$$

This formula shows that the observed signal can be thought of as a result of heterodyning (mixing) of the unperturbed probe field $\mathcal{E}_0$ with the field of scattering $\mathcal{E}_1$. Equation (3) also shows that, for sufficiently large dimensions of the detector ($l_{x,y}\gg\lambda=2\pi/k$), polarimetric signal u_1 represents *projection of the scattered field* (in the momentum space) onto the field of the probe beam. This means, in turn, that this signal is controlled by the fraction of the scattered field the distribution of which in space, to a certain extent, reproduces the field of the probe beam. Specifically, when the probe field represents a plane wave $\mathbf{E}_0\sim e^{i\mathbf{q}_0\mathbf{r}}$ with the wave vector $\mathbf{q}_0$, and the scattered field can be presented by a superposition of the plane waves $\mathbf{E}_1\sim\int d\mathbf{q}e^{i\mathbf{qr}}\mathbf{S}(\mathbf{q})$, the signal u_1 appears to be proportional to the component of the scattered field at the spatial frequency $\mathbf{q}_0$: $u_1\sim\mathbf{S}(\mathbf{q}_0)$.

Let us now calculate the scattered field $\mathcal{E}_1$.

III. POLARIMETRIC SIGNAL IN A MEDIUM WITH FLUCTUATING GYROTROPY

In this section, we consider scattering of a monochromatic light beam by the medium with randomly inhomogeneous (spatially fluctuating) gyrotropy. Temporal dependence of the polarimetric signal will be introduced to the derived equations by substituting static gyrotropy time dependent one. This "adiabatic" approximation is admissible because frequencies of the SNS signals are much lower than that of the optical beam. In this case, polarization of the medium $\mathbf{P}(\mathbf{r})$ can be expressed through the electric field $\mathbf{E}(\mathbf{r})$ as follows:

$$\mathbf{P}(\mathbf{r}) = i[\mathbf{E}(\mathbf{r})\mathbf{G}(\mathbf{r})] = i\mathbf{E}(\mathbf{r}) \times \mathbf{G}(\mathbf{r}), \tag{4}$$

where $\mathbf{G}(\mathbf{r})$ is the spatially dependent gyration vector. At this stage of our treatment, we assume the gyration vector to be time independent. Then, Maxwell's equations for the electromagnetic field in the medium can be reduced to the form

$$\Delta\mathbf{E} + k^2\mathbf{E} = -4\pi k^2\mathbf{P} - 4\pi \text{ grad div } \mathbf{P}, \quad k \equiv \frac{\omega}{c}. \tag{5}$$

We will search for a solution of this equation in the form of series in powers of $\mathbf{G}(\mathbf{r})$. The zeroth-order term $\mathbf{E_0}(\mathbf{r})$ represents the probe beam field that we consider to be known. The first-order term $\mathbf{E_1}(\mathbf{r})$ corresponds to the single-scattering approximation which is sufficient for our consideration. This term satisfies the equation

$$\begin{aligned}\Delta\mathbf{E_1} + k^2\mathbf{E_1} = &-4\pi i k^2\mathbf{E_0}(\mathbf{r}) \times \mathbf{G}(\mathbf{r})\\ &-4\pi i \text{ grad div } \mathbf{E_0}(\mathbf{r}) \times \mathbf{G}(\mathbf{r}).\end{aligned} \tag{6}$$

Solution of this equation can be expressed in terms of Green's function $\Gamma(\mathbf{r}) = -\exp(ikr)/4\pi r$ of the Helmholtz equation $[\Delta + k^2]\Gamma(\mathbf{r}) = \delta(\mathbf{r})$:

$$\begin{aligned}\mathbf{E_1}(\mathbf{r}) = i\int &\frac{\exp(ik|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|}[k^2\mathbf{E_0}(\mathbf{r}') \times \mathbf{G}(\mathbf{r}')\\ &+ \text{ grad div } \mathbf{E_0}(\mathbf{r}') \times \mathbf{G}(\mathbf{r}')]d^3\mathbf{r}'.\end{aligned} \tag{7}$$

Let the sample [the region where $\mathbf{G}(\mathbf{r})$ is nonzero] be placed in the vicinity of the origin of our coordinate system x,y,z. Let the photosensitive surface of the polarimetric detector be parallel to the xy plane and the detector itself be set at $z = L$, with L being large compared with the sample dimensions. Then, as seen from Eq. (7), the scattered field can be presented as a sum of two contributions:

$$\mathbf{E_1}(\mathbf{r}) = \mathbf{E_1^1}(\mathbf{r}) + \mathbf{E_1^2}(\mathbf{r}), \tag{8}$$

$$\begin{aligned}\mathbf{E_1^1}(\mathbf{r}) &\equiv \frac{ik^2}{L}\int \exp(ik|\mathbf{r}-\mathbf{r}'|)\mathbf{E_0}(\mathbf{r}') \times \mathbf{G}(\mathbf{r}')d^3\mathbf{r}',\\ \mathbf{E_1^2}(\mathbf{r}) &\equiv \frac{i}{L}\int \exp(ik|\mathbf{r}-\mathbf{r}'|) \text{ grad div } \mathbf{E_0}(\mathbf{r}')\\ &\quad\times \mathbf{G}(\mathbf{r}')d^3\mathbf{r}' = \frac{1}{k^2} \text{ grad div } \mathbf{E_1^1}(\mathbf{r}).\end{aligned}$$

We will concentrate on calculating the part $\mathbf{E_1^1}(\mathbf{r})$ of the scattered field because, in what follows, we will need this field at small scattering angles and, in this case, as it can be directly checked, only $\mathbf{E_1^1}(\mathbf{r})$ is of importance.

We take the probe beam in the form of Eq. (1). We need this field in two substantially separated spatial regions: first, in Eq. (3) at large values of $z \sim L$ and, second, in Eq. (8) at relatively small values of z within the sample. Calculation for $z \sim L$ shows that the field $\mathcal{E}_0$ entering Eq. (3) has the form

$$\begin{aligned}\begin{pmatrix}\mathcal{E}_{x0}(x,y,L)\\ \mathcal{E}_{y0}(x,y,L)\end{pmatrix} &= \begin{pmatrix}\cos\phi\\ \sin\phi\end{pmatrix}\sqrt{\frac{8W}{c}}\frac{k}{QL}\\ &\quad\times \sin\left[kL - \omega t + \frac{k[x^2+y^2]}{2L}\right]\\ &\quad\times \exp\left[-\frac{k^2(x^2+y^2)}{Q^2L^2}\right].\end{aligned} \tag{9}$$

While deriving this expression, we assumed that $L > z_c \equiv 4\pi w^2/\lambda$ (z_c is the Rayleigh length). Using this assumption, one has to accurately separate the real and imaginary parts in argument of the exponential in Eq. (1): the real part enters the exponential factor in Eq. (9) and describes the increasing beam diameter at large L, while the imaginary part yields the last term of the argument of the sin function and describes phase distribution of the optical field in the plane of the photodetector.

In what follows, the length of the sample l_s is supposed to be smaller than the Rayleigh length z_c. In the limit $|z| < z_c$, Eq. (1) can be simplified:

$$\begin{aligned}\mathbf{E_0}(\mathbf{r}) = &e^{i[kz-\omega t]}Q\sqrt{\frac{8W}{c}}\frac{(\cos\phi, \sin\phi, 0)}{2}\\ &\times \exp\left[-\frac{Q^2(x^2+y^2)}{4}\right], \quad z < z_c.\end{aligned} \tag{10}$$

Using this relationship, one can calculate the scattered field $\mathbf{E_1^1}(\mathbf{r})$ [Eq. (8)] and obtain, for real parts of $\mathcal{E}_{x1}$ and $\mathcal{E}_{y1}$ entering Eq. (3), the following expression:

$$\begin{aligned}\begin{pmatrix}\mathcal{E}_{x1}\\ \mathcal{E}_{y1}\end{pmatrix} &= \begin{pmatrix}-\sin\phi\\ \cos\phi\end{pmatrix}\sqrt{\frac{2W}{c}}\frac{Qk^2}{L}\\ &\quad\times \int \sin[k|\mathbf{r}-\mathbf{r}'| + kz' - \omega t]\\ &\quad\times \exp\left[-\frac{Q^2(x'^2+y'^2)}{4}\right]G_z(\mathbf{r}')d^3\mathbf{r}'.\end{aligned} \tag{11}$$

Using Eq. (3) and explicit expressions (9) and (11) for the probe $\mathcal{E}_0$ and scattered $\mathcal{E}_1$ fields, we can calculate the polarimetric signal. While averaging the product $\mathcal{E}_{x0}\mathcal{E}_{x1}$ over the optical period, we come to the integral

$$\begin{aligned}\frac{\omega}{\pi}\int_0^{2\pi/\omega}\mathcal{E}_{0x}\mathcal{E}_{1x}\,dt &\sim \frac{\omega}{\pi}\int_0^{2\pi/\omega}\sin[k|\mathbf{r}-\mathbf{r}'| + kz' - \omega t]\\ &\quad\times \sin\left[kL - \omega t + \frac{k[x^2+y^2]}{2L}\right]\\ &= \cos k\left[z' + |\mathbf{r}-\mathbf{r}'| - L - \frac{x^2+y^2}{2L}\right].\end{aligned} \tag{12}$$

The same is obtained for $\mathcal{E}_{y0}\mathcal{E}_{y1}$. Now, Eq. (3) gives

$$u_1 = -\frac{4Wk^3\sin[2\phi]}{cL^2}\int_{-l_x}^{l_x}dx\int_{-l_y}^{l_y}dy\exp\left[-\frac{k^2(x^2+y^2)}{Q^2L^2}\right] \times\int\cos k\left[z'+|\mathbf{r}-\mathbf{r}'|-L-\frac{x^2+y^2}{2L}\right] \times\exp\left[-\frac{Q^2(x'^2+y'^2)}{4}\right]G_z(\mathbf{r}')d^3\mathbf{r}', \tag{13}$$

with $\mathbf{r}=(x,y,L)$ and $\mathbf{r}'=(x',y',z')$. The external integration over $dxdy$ runs over the detector sensitive area, and, therefore, $|x|,|y|<l_{x,y}\ll L$. We assume that dimensions of the detector $l_{x,y}$ exceed the size $L\lambda/2\pi w$ of the probe beam spot at the detector [see Eq. (9)]. Then, x and y can be estimated as $|x|,|y|\sim L\lambda/2\pi w$. The internal integration $d\mathbf{r}'$ runs over the irradiated volume of the sample. For this reason, $x',y'\sim w$ and z' is of the order of the sample length l_s. Taking into account that $L\lambda/2\pi w,w,l_s\ll L$, we obtain the following expansion for the factor $|\mathbf{r}-\mathbf{r}'|$:

$$|\mathbf{r}-\mathbf{r}'|\approx L+\frac{x^2+y^2}{2L}+\frac{x'^2+y'^2}{2L}-\frac{xx'+yy'}{L}-z'. \tag{14}$$

Note that the term $\sim z'^2$ vanishes. Further estimates show that the term $(x'^2+y'^2)/2L$ can be omitted because, in our case, $k(x'^2+y'^2)/2L<\pi/4$ and, finally, we have

$$|\mathbf{r}-\mathbf{r}'|\approx L+\frac{x^2+y^2}{2L}-z'-\frac{xx'+yy'}{L}. \tag{15}$$

Using this formula, we can evaluate the product of the cosine functions in Eq. (13) as

$$\cos k\left[z'+|\mathbf{r}-\mathbf{r}'|-L-\frac{x^2+y^2}{2L}\right]=\cos k\left[\frac{xx'+yy'}{L}\right]. \tag{16}$$

As was mentioned above, the detector dimensions are assumed to be greater than the size of the probe beam spot: $l_{x,y}>L\lambda/2\pi w$. This allows one to extend integration over the detector surface in Eq. (13) to infinity, $|l_{x,y}|\to\infty$, and to calculate all integrals using the formula

$$\int dx\exp[-\alpha x^2+i\beta x]=\sqrt{\frac{\pi}{\alpha}}\exp\left(-\frac{\beta^2}{4\alpha}\right). \tag{17}$$

For example, the integral with cosine function in Eq. (16) (we denote it I_1) can be calculated as follows:

$$\begin{aligned}I_1 &= \int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\,\exp\left[-\frac{k^2(x^2+y^2)}{Q^2L^2}\right]\cos k\left[\frac{xx'+yy'}{L}\right]\\ &= \mathrm{Re}\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\exp\left[-\frac{k^2(x^2+y^2)}{Q^2L^2}+ik\frac{xx'+yy'}{L}\right]\\ &= \mathrm{Re}\int_{-\infty}^{\infty}dx\,\exp\left[-\frac{k^2x^2}{Q^2L^2}+ik\frac{xx'}{L}\right]\\ &\quad\times\int_{-\infty}^{\infty}dy\,\exp\left[-\frac{k^2y^2}{Q^2L^2}+ik\frac{yy'}{L}\right]\\ &= \frac{\pi Q^2L^2}{k^2}\exp\left(-\frac{[x'^2+y'^2]Q^2}{4}\right).\end{aligned} \tag{18}$$

Substituting Eq. (18) into Eq. (13), we obtain the following expression for the polarimetric signal:

$$u_1=-\frac{4Wk\pi Q^2\sin[2\phi]}{c}\times\int_V\exp\left[-\frac{Q^2(x'^2+y'^2)}{2}\right]G_z(\mathbf{r}')d^3\mathbf{r}'. \tag{19}$$

Recall that this formula is valid if the sample length l_s is smaller than the Rayleigh length, $l_s<z_c$ [see definition of the Rayleigh length after Eq. (9)] and the probe beam spot is smaller than the detector photosensitive area, $l_{x,y}\gg L\lambda/2\pi w$. It is seen from Eq. (19) that the polarimetric signal is, in fact, proportional to the z component of the gyration averaged over the irradiated volume of the sample, as is usually implied intuitively.

Equation (19) allows one to obtain the expression for the magnetization noise power spectrum observed in the SNS. In this case, $\mathbf{G}(\mathbf{r})$ is proportional to instantaneous spontaneous magnetization of the sample randomly fluctuating both in space and in time. If characteristic frequencies of this field are much lower than the optical frequency ω, one can use Eq. (19) for calculating the random polarimetric signal by substituting $\mathbf{G}(\mathbf{r})\to\mathbf{G}(\mathbf{r},t)$. The noise power spectrum $\mathcal{N}(\nu)$ is defined as the Fourier transform of the correlation function of the polarimetric signal. Using Eq. (19), the noise power spectrum $\mathcal{N}(\nu)$ can be expressed in terms of the spatiotemporal correlation function of the gyrotropy $\mathbf{G}(\mathbf{r},t)$:

$$\begin{aligned}\mathcal{N}(\nu) &= \int dt\langle u_1(t)u_1(0)\rangle\,e^{i\nu t}\,\frac{16W^2k^2\pi^2Q^4\sin^2[2\phi]}{c^2}\\ &\quad\times\int dt e^{i\nu t}\int_V d^3\mathbf{r}\int_V d^3\mathbf{r}'\\ &\quad\times\exp\left[-\frac{Q^2(x'^2+y'^2+x^2+y^2)}{2}\right]\\ &\quad\times\langle G_z(\mathbf{r}',0)G_z(\mathbf{r},t)\rangle.\end{aligned} \tag{20}$$

To calculate the correlation function $\langle G_z(\mathbf{r}',0)G_z(\mathbf{r},t)\rangle$ entering Eq. (20), one should specify a particular model of the gyrotropic medium. The example of such a model (the model of independent paramagnetic atoms with fluctuating magnetization) will be described in Sec. V. In the next section, we will calculate the polarimetric signal provided by the auxiliary tilted beam that produces the scattered field but does not irradiate the detector (see Fig. 1).

IV. DETECTING THE SCATTERED FIELD OF THE TILTED BEAM

Let the sample be illuminated by an auxiliary light beam (AB) propagating at the angle Θ with respect to the main probe beam Fig. 1. Note that the AB does not hit the detector, but the scattered field of this beam may provide additional contribution to the detected polarimetric signal, and our goal now is to calculate the value of this contribution.

The calculation can be performed in the same way as in the previous section with the following changes. The scattered field is calculated using Eq. (8), with the field $\mathbf{E_0}(\mathbf{r})$ replaced by $\mathbf{E}_0^{\mathbf{t}}(\mathbf{r})$, where $\mathbf{E}_0^{\mathbf{t}}(\mathbf{r})$ represents the field of the auxiliary (tilted) beam. The field $\mathbf{E}_0^{\mathbf{t}}(\mathbf{r})$ can be obtained by rotating $\mathbf{E_0}(\mathbf{r})$ by the angle Θ around the axis $(\cos\phi, \sin\phi, 0)$ parallel to the direction of polarization of the probe beam: [17]

$$\mathbf{E}_\mathbf{0}^\mathbf{t}(\mathbf{r}) = M\mathbf{E}_0(M\mathbf{r}). \tag{21}$$

Here, the matrix M is defined as

$$\begin{aligned} M = R(-\phi)H(\Theta)R(\phi) &= \begin{pmatrix} \cos\Theta\sin^2\phi + \cos^2\phi & [1-\cos\Theta]\sin\phi\cos\phi & -\sin\phi\sin\Theta \\ [1-\cos\Theta]\sin\phi\cos\phi & \cos\Theta\cos^2\phi + \sin^2\phi & \cos\phi\sin\Theta \\ \sin\Theta\sin\phi & -\sin\Theta\cos\phi & \cos\Theta \end{pmatrix} \\ &= \begin{pmatrix} 1-\frac{1}{2}\Theta^2\sin^2\phi & \frac{1}{2}\Theta^2\sin\phi\cos\phi & -\Theta\sin\phi \\ \frac{1}{2}\Theta^2\sin\phi\cos\phi & 1-\frac{1}{2}\Theta^2\cos^2\phi & \Theta\cos\phi \\ \Theta\sin\phi & -\Theta\cos\phi & 1-\frac{1}{2}\Theta^2 \end{pmatrix} + O(\Theta^3). \end{aligned} \tag{22}$$

Therefore, the field $\mathbf{E}_\mathbf{0}^\mathbf{t}(\mathbf{r})$ is defined by the expression

$$\mathbf{E}_\mathbf{0}^\mathbf{t}(\mathbf{r}) = Q\sqrt{\frac{2W_t}{c}}(\cos\phi, \sin\phi, 0)\ \exp i[kZ(\mathbf{r}) - \omega t]\ \exp\left[-\frac{Q^2[X^2(\mathbf{r}) + Y^2(\mathbf{r})]}{4}\right], \tag{23}$$

where

$$\begin{pmatrix} X(\mathbf{r}) \\ Y(\mathbf{r}) \\ Z(\mathbf{r}) \end{pmatrix} \equiv \begin{pmatrix} \cos\Theta\sin^2\phi + \cos^2\phi & [1-\cos\Theta]\sin\phi\cos\phi & -\sin\phi\sin\Theta \\ [1-\cos\Theta]\sin\phi\cos\phi & \cos\Theta\cos^2\phi + \sin^2\phi & \cos\phi\sin\Theta \\ \sin\Theta\sin\phi & -\sin\Theta\cos\phi & \cos\Theta \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} \delta x \\ \delta y \\ \delta z \end{pmatrix} \tag{24}$$

with $\mathbf{r} = (x, y, z)$. We denote by W_t intensity of the AB and take into account its possible spatial shift $(\delta x, \delta y, \delta z)$. Substituting $\mathbf{E}_\mathbf{0}^\mathbf{t}(\mathbf{r})$ [Eq. (23)] into Eq. (8) instead of $\mathbf{E}_\mathbf{0}(\mathbf{r})$, one can obtain the following expression for the scattered field produced by the AB:

$$\begin{pmatrix} \mathcal{E}_{1x}^t \\ \mathcal{E}_{1y}^t \end{pmatrix} = \begin{pmatrix} -\sin\phi \\ \cos\phi \end{pmatrix} \sqrt{\frac{2W_t}{c}} \frac{Qk^2}{L} \int \sin[k|\mathbf{r} - \mathbf{r}'| + kZ' - \omega t] \exp\left[-\frac{Q^2(X'^2 + Y'^2)}{4}\right] G_z(\mathbf{r}') d^3\mathbf{r}', \tag{25}$$

where $X' = X(\mathbf{r}')$, $Y' = Y(\mathbf{r}')$, and $Z' = Z(\mathbf{r}')$, with the functions $X(\mathbf{r}')$, $Y(\mathbf{r}')$, $Z(\mathbf{r}')$ defined by Eq. (24) with substitution $x, y, z \to x', y', z'$. This formula has the same sense as Eq. (11); for clarity we supply components of the scattered field by superscript t. Taking into account this replacement, one can get the relationship for the polarimetric signal produced by the AB [instead of Eq. (13)]:

$$\begin{aligned} u_1^t = &-\frac{4\sqrt{WW_t}k^3\sin[2\phi]}{cL^2} \int_{-l_x}^{l_x} dx \int_{-l_y}^{l_y} dy \exp\left[-\frac{k^2(x^2+y^2)}{Q^2L^2}\right] \\ &\times \int \cos\ k\left[|\mathbf{r} - \mathbf{r}'| + Z' - L - \frac{x^2+y^2}{2L}\right] \exp\left[-\frac{Q^2(X'^2 + Y'^2)}{4}\right] G_z(\mathbf{r}') d^3\mathbf{r}'. \end{aligned} \tag{26}$$

Calculation of integrals can be made as in the previous section, and the final result for the polarimetric signal produced by the AB is

$$u_1^t = -\frac{4\sqrt{WW_t}k\pi Q^2\sin[2\phi]}{c} \int_V \cos k[z' - Z'] \exp\left[-\frac{Q^2(x'^2 + y'^2 + X'^2 + Y'^2)}{4}\right] G_z(\mathbf{r}') d^3\mathbf{r}', \tag{27}$$

where $\mathbf{r}' = (x', y', z')$ and

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \begin{pmatrix} \cos\Theta\sin^2\phi + \cos^2\phi & [1-\cos\Theta]\sin\phi\cos\phi & -\sin\phi\sin\Theta \\ [1-\cos\Theta]\sin\phi\cos\phi & \cos\Theta\cos^2\phi + \sin^2\phi & \cos\phi\sin\Theta \\ \sin\Theta\sin\phi & -\sin\Theta\cos\phi & \cos\Theta \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} + \begin{pmatrix} \delta x \\ \delta y \\ \delta z \end{pmatrix}. \tag{28}$$

One can see that u_1^t is proportional to overlap of the two beams and vanishes at large shifts $\delta x, \delta y, \delta z$. The trigonometric factor $\cos k[z - Z]$, in fact, singles out the harmonic of the gyrotropy with the spatial frequency equal to the difference between the wave vectors of the two beams. Total signal in the presence of two beams is the sum of Eqs. (19) and (27): $u_1 + u_1^t$. Recall that the angle Θ should not be too large; otherwise, one should take into account the component $\mathbf{E}_\mathbf{1}^\mathbf{2}(\mathbf{r})$ in Eq. (8).

V. NOISE SIGNAL IN THE TWO-BEAM CONFIGURATION

The noise signal produced by the two beams in the configuration of Fig. 1 is calculated as the Fourier transform of the correlation function of the total polarimetric signal $u = u_1 + u_1^t$. It consists of three terms:

$$\mathcal{N}_t(\nu) = \int dt e^{i\nu t} \langle u(0)u(t)\rangle = \int dt e^{i\nu t} \big[\langle u_1(0)u_1(t)\rangle + 2\langle u_1(0)u_1^t(t)\rangle + \langle u_1^t(0)u_1^t(t)\rangle\big]. \tag{29}$$

Using Eqs. (19) and (27), one can write the expressions for each of them. The first term has been already calculated and is given by Eq. (20). For the correlator entering the last term, we have

$$\langle u_1^t(0)u_1^t(t)\rangle = \frac{16WW_tk^2\pi^2Q^4\sin^2[2\phi]}{c^2}\int_V d^3\mathbf{r}\int_V d^3\mathbf{r}'\cos k[z-Z]\cos k[z'-Z'] \times \exp\left[-\frac{Q^2(X^2+Y^2+x^2+y^2+X'^2+Y'^2+x'^2+y'^2)}{4}\right]\times\langle G_z(\mathbf{r}',0)G_z(\mathbf{r},t)\rangle, \tag{30}$$

where $x,y,z\to\mathbf{r}$ and X,Y,Z are defined by Eq. (28). Finally, the cross correlator $\langle u_1^t(0)u_1(t)\rangle$ can be written as

$$\langle u_1^t(0)u_1(t)\rangle = \frac{16W\sqrt{WW_t}k^2\pi^2Q^4\sin^2[2\phi]}{c^2}\int_V d^3\mathbf{r}\int_V d^3\mathbf{r}'\cos k[z-Z] \times \exp\left[-\frac{Q^2(X^2+Y^2+x^2+y^2)}{4}-\frac{Q^2(x'^2+y'^2)}{2}\right]\times\langle G_z(\mathbf{r}',0)G_z(\mathbf{r},t)\rangle. \tag{31}$$

Consider now the physical sense of different factors entering Eqs. (20), (30), and (31).

The *exponential factor* reduces the region of integration down to the region of overlap of the two beams. If Θ is not too large and $l_s\Theta < w$, this region is close to the "beam volume within the sample." In this case, the exponential factor can be calculated at $X=x, Y=y, Z=z, X'=x', Y'=y', Z'=z'$. Note that it is rather difficult to satisfy the condition $l_s\Theta < w$ in a real experiment. For this reason, the overlapping factor may considerably reduce contribution of the AB to the polarimetric signal.

It is noteworthy that, as follows from Eq. (31), contribution of the auxiliary beam to the detected signal is controlled by the volume of overlap of the two beams, which is getting smaller with increasing Θ. Though the value of this contribution, under these conditions, becomes smaller, spatial resolution of this technique, which can be evidently used for tomographic purposes, can be substantially improved.

The *trigonometric factor* at small angles Θ is controlled by the difference between wave vectors of the two beams because the cosine argument can be evaluated as $z-Z=[\cos\phi y-\sin\phi x]\Theta$.

The *correlation function* $\langle G_z(\mathbf{r}',0)G_z(\mathbf{r},t)\rangle$ is determined by a particular model of the gyrotropic medium. For homogeneous media, it depends on the difference $\mathbf{r}-\mathbf{r}'$ of the spatial arguments. For the model of independent spins described below, $\langle G_z(\mathbf{r}',0)G_z(\mathbf{r},t)\rangle \sim \delta(\mathbf{r}-\mathbf{r}')e^{-|t|/\tau}\cos\omega_0 t$.

Thus, the integrals entering Eqs. (20), (30), and (31) can be calculated for any particular model of the gyrotropic medium. In the next section, we will present calculations for the model of independent paramagnetic particles (spins). Still, the following general remark should be made. Let the beam waist $4w$ and the sample length l_s be much greater than the gyrotropy correlation radius R_c and spatial period $2\pi/k\Theta$ related to the difference of wave vectors of the two beams: $4w, l_s \gg R_c, 2\pi/k\Theta$. Then, one can substitute variables in the integrals entering Eqs. (20), (30), and (31) in the following way, $\mathbf{r},\mathbf{r}'\to\mathbf{R}\equiv\mathbf{r}-\mathbf{r}', \mathbf{R}'\equiv\mathbf{r}+\mathbf{r}'$, and take advantage of the fact that the correlator $\langle G_z(\mathbf{r}',0)G_z(\mathbf{r},t)\rangle$ depends on the difference of its arguments:

$$\langle G_z(\mathbf{r}',0)G_z(\mathbf{r},t)\rangle \equiv K(\mathbf{r}-\mathbf{r}',t). \tag{32}$$

Then, the integral over $\mathbf{R}\equiv\mathbf{r}-\mathbf{r}'$ in Eq. (20) can be estimated as the average of $K(\mathbf{R},t)$ over the irradiated volume of the sample V_b. The integration over $\mathbf{R}'\equiv\mathbf{r}+\mathbf{r}'$ gives this volume itself, and we obtain

$$\begin{aligned}\mathcal{N}(\nu) &= \frac{16W^2k^2\pi^2Q^4\sin^2[2\phi]}{c^2}\int dt\, e^{i\nu t}\\ &= \int_V d\mathbf{r}d\mathbf{r}'\exp\left[-\frac{Q^2(x'^2+y'^2+x^2+y^2)}{2}\right]\\ &\quad\times K(\mathbf{r}-\mathbf{r}',t)\\ &\sim \frac{W^2l_s\sin^2[2\phi]}{S}\int dt\, e^{i\nu t}\int_{V_b} d\mathbf{R}\, K(\mathbf{R},t).\end{aligned} \tag{33}$$

Here, we denote the cross-section area of the beam by $S\equiv 4\pi w^2$ and take into account that $w=1/Q$ and that the irradiated volume of the sample is $V_b = Sl_s$, where l_s is the sample length. We come to the known result that the noise power signal is proportional to the sample length and inversely proportional to the beam cross section [1,2,4].

The correlation function Eq. (30) can be estimated in a similar way. If Θ is not too large, then the arguments of the cosine functions can be evaluated as $z-Z=[\cos\phi y-\sin\phi x]\Theta$ and $z'-Z'=[\cos\phi y'-\sin\phi x']\Theta$. Therefore, one can represent the product of the cosine functions in Eq. (30) as

$$\begin{aligned}&\cos k[z-Z]\cos k[z'-Z']\\ &= \frac{1}{2}\cos\{k\Theta[(y-y')\cos\phi-(x-x')\sin\phi]\}\\ &\quad+\frac{1}{2}\cos\{k\Theta[(y+y')\cos\phi-(x+x')\sin\phi]\}.\end{aligned}$$

Note that the difference $\Delta\mathbf{k}$ between the wave vector of the two beams for small Θ has only x and y components: $\Delta\mathbf{k}=k\Theta(-\sin\phi,\cos\phi,0)$. Therefore, this relationship after substitution of variables $\mathbf{r},\mathbf{r}'\to\mathbf{R}=\mathbf{r}-\mathbf{r}',\mathbf{R}'=\mathbf{r}+\mathbf{r}'$ takes the form

$$\cos k[z-Z]\cos k[z'-Z'] = \frac{1}{2}\cos(\Delta\mathbf{k},\mathbf{R})+\frac{1}{2}\cos(\Delta\mathbf{k},\mathbf{R}').$$

Recall that our treatment is valid when w is large enough ($\Delta k w > 2\pi$). In this case, the integral $\int_{V_b} d\mathbf{R}'\cos(\Delta\mathbf{k},\mathbf{R}')\sim 0$, and we come to the conclusion that the correlation function

Eq. (30) can be estimated as follows:

$$\begin{aligned}&\langle u_1^t(0)u_1^t(t)\rangle\\ &\sim WW_t Q^4 \sin^2[2\phi]\int_{V_b} d\mathbf{R}d\mathbf{R}'\ K(\mathbf{R},t)\cos(\Delta\mathbf{k},\mathbf{R})\\ &\sim \frac{WW_t \sin^2[2\phi]l_s}{S}\int_{V_b} d\mathbf{R}\ K(\mathbf{R},t)\cos(\Delta\mathbf{k},\mathbf{R}).\end{aligned} \tag{34}$$

Thus, contribution of the AB to the noise signal is proportional to the Fourier transform of the correlation function of gyrotropy at spatial frequency equal to the difference of the wave vectors of the two beams ($\Delta\mathbf{k}$).

Therefore, by measuring dependence of the noise signal, in the two-beam configuration, on the angle between the beams (in fact, on Δk) and using the inverse Fourier transform, one can restore spatial dependence of the gyrotropy correlation function $K(\mathbf{R},t)$. Recall that in the conventional spin-noise spectroscopy, only temporal dependence of this correlation function averaged over the irradiated volume of the sample is revealed.

Similarly, it can be shown that, under these conditions, contribution of the cross correlator Eq. (31) is relatively small.

VI. THE MODEL OF INDEPENDENT SPINS

In this model, the random field of gyrotropy $G_z(\mathbf{r})$ has the form

$$G_z(\mathbf{r}) = \sum_{i=1}^{N} g_i(t)\delta(\mathbf{r}-\mathbf{r}_i), \tag{35}$$

thus corresponding to N paramagnetic particles (spins) randomly distributed over the volume of the medium with some average density $\sigma \equiv N/V$, where V is the total volume of the system. We assume that $g_i(t)$ is proportional to the z component of magnetization of the ith particle. The polarimetric signal can be calculated using Eq. (19):

$$\begin{aligned}u_1 = u_1(t) = &-\frac{4Wk\pi Q^2\sin[2\phi]}{c}\\ &\times\int_V \exp\left[-\frac{Q^2(x'^2+y'^2)}{2}\right]\sum_i g_i(t)\delta(\mathbf{r}'-\mathbf{r_i})d^3\mathbf{r}'.\end{aligned} \tag{36}$$

Let us calculate polarimetric signal u_{10} for the sample in which all magnetizations $g_i(t)$ are constant and the same: $g_i(t) = g_0 = \text{const}$. This corresponds to a paramagnet in a high magnetic field at low temperature. In this case, Eq. (36) gives

$$u_{10} = -\frac{8Wkg_0\sigma l_s\pi^2\sin[2\phi]}{c}. \tag{37}$$

We will see below that the quantity u_{10} provides us a convenient scale. Let us now consider the gyrotropic medium with the quantities g_i changing randomly in a stationary way with the correlation function $\langle g_i(t)g_k(t')\rangle = \delta_{ik}K(t-t')$ [which should be distinguished from the spatiotemporal correlation function of Eq. (32)] and calculate, for this model, the noise power spectrum using Eq. (20). We have

$$\begin{aligned}&\langle G_z(\mathbf{r}',0)G_z(\mathbf{r},t)\rangle\\ &= \frac{1}{V}\sum_i\int d^3\mathbf{r_i}\langle g_i(0)g_i(t)\rangle\delta(\mathbf{r}'-\mathbf{r_i})\delta(\mathbf{r}-\mathbf{r_i})\\ &= \delta(\mathbf{r}-\mathbf{r}')\sigma K(t),\end{aligned} \tag{38}$$

and, consequently,

$$\mathcal{N}(\nu) = \frac{16W^2k^2\pi^3Q^2l_s\sigma\sin^2[2\phi]}{c^2}\int dt\ e^{i\nu t}K(t). \tag{39}$$

If we accept for the beam area the expression $S = 4\pi w^2$, then $Q^2 = 1/w^2 = 4\pi/S$. Taking into account Eq. (37), we obtain the expression for the noise power spectrum:

$$\mathcal{N}(\nu) = \frac{u_{10}^2}{\sigma l_s S}\int dte^{i\nu t}\frac{\langle g(0)g(t)\rangle}{g_0^2}. \tag{40}$$

Note that $\sigma l_s S \equiv N_b$ is the number of spins in the irradiated volume of the sample.

In the simplest case, each paramagnetic particle of the gyrotropic medium can be associated with the effective spin 1/2. Then, the total magnetization can be expressed as $g_0^2 = (g\beta)^2/4$ (here, g is the effective g factor and β is the Bohr magneton). In the presence of the transverse magnetic field B_x, the correlator $\langle g(0)g(t)\rangle$ can be calculated using the following chain of relationships:

$$\begin{aligned}\langle g(0)g(t)\rangle &= \frac{(g\beta)^2}{2}\,\text{Sp}\,[S_zS_z(t)+S_z(t)S_z]\rho_{\text{eq}},\\ S_z(t) &= e^{-i\omega_0tS_x}S_ze^{i\omega_0tS_x}\omega_0 \equiv \frac{g\beta B_x}{\hbar}.\end{aligned} \tag{41}$$

Here, ρ_{eq} is the density matrix of the two-level system representing our effective spin 1/2. If the temperature is high enough ($kT \gg g\beta B_x$), the density matrix can be taken constant, $\rho_{\text{eq}} = \hat{I}/2$ ($\hat{I}$ is the unit matrix), and we obtain

$$\begin{aligned}\langle g(0)g(t)\rangle &= \frac{(g\beta)^2}{4}\,\text{Sp}\,[S_zS_z(t)+S_z(t)S_z]\\ &= \frac{(g\beta)^2}{4}\,\cos\omega_0t \to \frac{(g\beta)^2}{4}\,e^{-|t|/\tau}\cos\omega_0t\\ &\Rightarrow \frac{\langle g(0)g(t)\rangle}{g_0^2} = e^{-|t|/\tau}\cos\omega_0t.\end{aligned} \tag{42}$$

Here we introduce phenomenologically the transverse relaxation time τ. So, for the noise power spectrum, we have

$$\mathcal{N}(\nu) = \frac{u_{10}^2\tau}{N_b}\left[\frac{1}{1+(\omega_0+\nu)^2\tau^2}+\frac{1}{1+(\omega_0-\nu)^2\tau^2}\right]. \tag{43}$$

The root-mean-square value of the polarimetric noise is given by the relationship

$$\langle\delta u^2\rangle = \frac{1}{2\pi}\int\mathcal{N}(\nu)d\nu = \frac{u_{10}^2}{N_b}. \tag{44}$$

In a similar way, one can calculate the power spectrum of the polarimetric noise in the presence of the auxiliary beam AB. Using Eq. (37) for the correlation function and Eqs. (20), (30), and (31), we obtain

$$\langle u(0)u(t)\rangle = u_{10}^2 \frac{e^{-|t|/\tau}\cos\omega_0 t}{N_b} \times \left\{1 + 2\sqrt{\frac{W_t}{W}}\exp\left[-\frac{k^2\Theta^2}{4Q^2}\right] + \frac{W_t}{W}\frac{1}{2}\left(1+\exp\left[-\frac{k^2\Theta^2}{Q^2}\right]\right)\right\}. \quad (45)$$

If $2\pi/k = 1\ \mu$m, $\Theta \sim 0.1$, and $1/Q \sim 30\ \mu$m, the exponential factors can be omitted, and simplified expressions for the correlation function and for the noise power spectrum acquire the form

$$\langle u(0)u(t)\rangle = \frac{u_{10}^2}{N_b}\left[1+\frac{W_t}{2W}\right]e^{-|t|/\tau}\cos\omega_0 t, \quad (46)$$

$$\mathcal{N}(\nu) = \frac{u_{10}^2\tau}{N_b}\left[1+\frac{W_t}{2W}\right] \times \left[\frac{1}{1+(\omega_0+\nu)^2\tau^2} + \frac{1}{1+(\omega_0-\nu)^2\tau^2}\right]. \quad (47)$$

It is seen from Eq. (47) that if $W_t \sim W$ then switching the auxiliary beam on leads to 50% increase of the noise power and, therefore, can be easily observed. Note once again that we assumed complete overlap of the two beams. Therefore, the contribution of the auxiliary beam to the noise power spectrum in real experiments, when this is not the case, may be somewhat smaller.

The above treatment was performed for the case of absence of any spatial correlation in the field of gyrotropy. The result of this assumption is the absence of any dependence of the noise signal on the angle Θ (at small Θ). In the presence of spatial correlation of the gyrotropy, the noise signal will decrease with Θ (with increasing $\Delta k = k\Theta$). Specifically, if the noise signal decreases, say, by a factor of 2 at an angle of $\Theta_{1/2}$, then the correlation radius of the gyration field R_c can be estimated as $R_c \sim [\Delta k_{1/2}]^{-1} \equiv [k\Theta_{1/2}]^{-1}$.

To estimate and probably to measure the gyrotropy field correlation function in the above arrangement, the tilted beam can be obtained using the same lens that is employed to focus the probe beam. For this purpose, the auxiliary beam propagating parallel to the probe and shifted from it by the distance D hits this lens. Then, the two beams will intersect after the lens in the vicinity of its focal plane at the angle D/f, where f is the focal length of the lens. By varying the distance D, one can change the above angle and observe changes in the noise signal needed to calculate the correlation function of the gyrotropy. The maximum (minimum) angle of beam convergence that can be obtained in this way is estimated as $\Theta_{\max} \approx R/f$ ($\Theta_{\min} \approx d/f$), where R is the lens radius and d is the diameter of the beams incident upon the lens. Taking $R = 25$ mm, $f = 50$ mm, and $d = 3$ mm, we obtain $\Theta_{\min} = 0.06$ rad and $\Theta_{\max} = 0.5$ rad. If the light wavelength is $\lambda = 1\ \mu$m, then the range of the correlation radii that can be covered in the above two-beam arrangement is $R_c^{\min} = \lambda/2\pi\Theta_{\max} \approx 0.3\ \mu$m and $R_c^{\max} = \lambda/2\pi\Theta_{\min} \approx 3\ \mu$m. Note that reduction of the noise signal with increasing Θ can also occur due to decreasing overlap of the beams. This effect is to be taken into account upon processing of the experimental data. However, for the samples thin enough compared with the Rayleigh length, the role of this factor may be insignificant (the Rayleigh length for the beams considered above lies in the range of 1 mm).

VII. DISCUSSION AND CONCLUSIONS

The polarization noise signal in SNS is known to be a result of mixing of the field scattered by fluctuating spins with that of the probe beam. The main goal of this paper was to present rigorous description of signal formation in SNS and to figure out whether the scattered field coming out of the sample practically isotropically can be used more efficiently in the SNS measurements. The sample with fluctuating spins is considered here as an inhomogeneous optical medium with its gyrotropy fluctuating both in time and in space. The noise signal arising due to heterodyning of the scattered light is calculated for a focused Gaussian beam in the single-scattering approximation. We show that, in real experiments, only a fraction of the scattered field that overlaps with the probe beam in the momentum space contributes to the detected signal. Therefore, a more efficient use of the scattered field, in spin-noise spectroscopy, can be achieved by increasing this overlap in the proper optical arrangement. Our calculations confirm the common assumption that the noise signal, in the conventional geometry of SNS, is proportional to the sample's gyrotropy spatially averaged over the irradiated volume. We also consider a two-beam geometry in which properties of the scattered light field are revealed in a much more pronounced way. It is shown that the additional signal produced by the auxiliary light beam, tilted with respect to the probe, is proportional to the Fourier transform of the gyrotropy correlation function at spatial frequency equal to the difference of wave vectors of the two beams. Accordingly, in the presence of spatial correlation of the gyrotropy, Fourier components at higher spatial frequencies will appear to be suppressed, and contribution of the auxiliary beam at larger angles between the beams will decrease. This effect can be used to investigate spatial correlation of spins related, e.g., to spin-spin interactions, motion of spin carriers, etc.

Our calculations also show that the additional signal, in the two-beam geometry, is produced only by the region of spatial overlap of the two beams. This means that the two-beam geometry allows one to realize the SNS-based 3D tomography with substantially higher spatial resolution than in the standard single-beam configuration [8,9]. The results of rigorous solution of the problem are presented here for the case of spatially uncorrelated gyrotropy with the "white" spectrum of the gyrotropy spatial fluctuations.

Note that calculations of the SNS signal were performed, in this paper, for a simplified optical arrangement with no lenses after the scattering volume. We believe that our conclusions, in their main features, will remain valid for standard optical schemes with dioptric elements. At the same time, we admit that, for some specific arrangements of focusing and collecting the fields on the detectors, the overall conditions of heterodyning may change and our conclusions will need to be corrected. We plan to analyze this issue more accurately elsewhere.

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