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PHYSICAL OPTICS

A Theory of Excitation of a Planar Semiconductor Optical Waveguide Using a Diffraction Grating: Single-Scattering Approximation

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Abstract—The problem of excitation of a totally reflecting planar optical waveguide using a coupling diffraction grating in the form of a periodic relief of the waveguide-layer thickness is solved within the single-scattering approximation. The polariton mode in the presence of a quantum well near the waveguide is considered. Based on the developed concepts, the following experimental features of the dependence of the intensity of radiation conducted in the waveguide layer on the angle of incidence of the excitation beam on the coupling diffraction grating are interpreted: the dependence on the mode number, the interference effects in the presence of two coupling diffraction gratings, and the influence of the lower substrate boundary on the thermal behavior of the waveguide structure.

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1. INTRODUCTION

In recent decades, layered structures obtained by molecular-beam epitaxy have been popular objects of spectroscopic studies. An important class of these structures includes combined GaAs/AlGaAs/InGaAs semiconductor structures, which are optical Bragg waveguides coupled with quantum wells (QWs) [1]. These structures are important because the polariton mode of an electromagnetic field therein exhibits a number of interesting phenomena: bimodal reflection spectrum (Rabi splitting) [1], parametric amplification [2], lasing [3], Bose–Einstein condensation of polaritons [4], light delay [5], etc.

However, the design of a combined Bragg waveguide–QW structure is fairly difficult from the technological point of view, because Bragg mirrors should be rather "thick" and consist of several tens of layers to provide high reflectivity. Therefore, it is an urgent problem to search for simpler structures with a strongly interacting electromagnetic optical mode and material resonance. In this context, a combined structure [6], similar to the above-described one but with a Bragg waveguide replaced by a totally reflecting (TR) waveguide [7–9], is of great interest.

The structure of a TR waveguide (a layer of high-reflectivity material inserted in a low-reflectivity medium) is extremely simple; however, a special device should be used to inject light into the TR waveguide [8]. It can be a prism glued on the TR waveguide (tunnel injection) [7, 9] or coupling diffraction grating formed on its surface [8–10]. As far as the possibility of using TR waveguides in compact optical data-processing devices is concerned, light injection using a diffraction grating is preferred because it does not violate the structure planarity. Moreover, one of the most widespread types of the aforementioned layered structures comprises epitaxial GaAs/AlGaAs/InGaAs semiconductor structures, for which ion etching technology capable of forming amplitude diffraction gratings on the surface has been developed.

Although light injection into planar waveguides using coupling diffraction gratings is well known [10, 11], we could find only one reference [12] where this problem was considered within the single-scattering approximation. In this study, we present such a consideration as applied to the calculation of the TR waveguide–QW combined structure. The reported solution makes it possible to find the electromagnetic-field amplitude in the TR waveguide at large distances from the light-injection point for specified parameters of the diffraction grating and waveguide.

We experimentally observed the dependence of the field intensity in the TR waveguide on the angle of incidence of the excitation laser beam with a fixed frequency onto the coupling diffraction grating. This dependence has a maximum when the angle at which the beam diffracted into the waveguide propagates is equal to the characteristic angle of one of the TR-waveguide modes. Based on these experimental conditions, we show in the theoretical part of this paper that the presence of a QW near the waveguide[1] may lead to a bimodal form of the above angular dependence. This phenomenon is similar to the splitting of the frequency dependence of the intensity of light reflection from the Bragg waveguide–QW structure in the strong-coupled mode [1]. The calculated intermediate results suggested a simple way to estimate the amplitude of the coupling diffraction grating.

In the experimental part of the study, we investigate a GaAs/AlGaAs semiconductor TR waveguide with coupling diffraction gratings formed by vacuum etching. Using the obtained solution of the electrodynamic problem, we interpret the difference in the excitation of different TR-waveguide modes, the interference effects arising upon excitation of a TR waveguide by two gratings, and the thermal behavior of the waveguide structure. In addition, we present the results of testing the proposed method for estimating the grating amplitude.

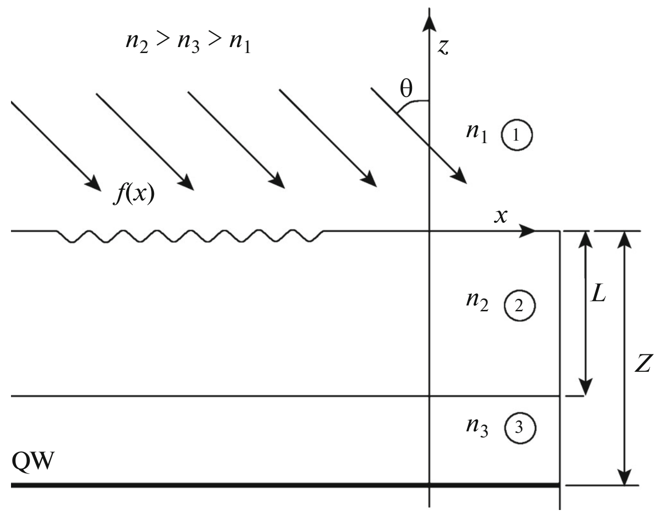

Fig. 1. TR waveguide with a coupling diffraction grating. Oblique arrows indicate a plane wave incident on the structure. The media are enumerated on the right and the corresponding refractive indices are presented. The narrow stripe at the bottom indicates the QW.

2. STATEMENT OF THE PROBLEM AND THE BOUNDARY CONDITIONS

We consider a structure comprising a TR waveguide with a coupling diffraction grating on its surface (Fig. 1) and state the problem as follows. Let a plane monochromatic wave with frequency ω be incident from the upper half-space, making angle θ with the structure. The scattered field arising in this case must be found. The used approximations will be formulated below during the solution of the above-stated problem, and this statement will be refined taking into account that we eventually would like to analyze the excitation of the TR waveguide formed by layer 2 (we assume that $n_2 > n_3 > n_1$). We will consider this structure as homogeneous along the y axis and perform the calculations for the case of TE waves, where the electric field has only the y component in the whole space.

This problem is difficult to solve because the interface between media 1 and 2 (Fig. 1) in the layered system under consideration is not planar but determined by a specified function $z = f(x)$, which can describe, in particular, the amplitude diffraction grating. We will consider the function $z = f(x)$ small in comparison with wavelength λ of the used light and sufficiently smooth. This corresponds to the following conditions imposed on $f(x)$:

$$n_2 k f(x) \ll 1, \quad n_2 f'(x) \ll 1, \tag{1}$$

[1] The interaction between a QW and the waveguide modes is due to the "evanescent tails" of the latter.

where $k \equiv 2\pi/\lambda = \omega/c$ and c is the speed of light. Below we will need for the Fourier transform F_q of this function:

$$F_q \equiv \frac{1}{2\pi}\int dx e^{-\imath qx} f(x), \quad f(x) = \int F_q e^{\imath qx} dq. \tag{2}$$

If function $f(x)$ corresponds to a cosine diffraction grating with period $b = 2\pi/s$ and length $2B$,

$$\begin{gathered} f(x) = \begin{cases} \varepsilon\cos(sx), \text{ if } x \in [-B, B] \\ 0, \text{ otherwise}, \end{cases} \\ F_q = \frac{\varepsilon}{2\pi}\left\{\frac{\sin(s-q)B}{s-q} + \frac{\sin(s+q)B}{s+q}\right\}. \end{gathered} \tag{3}$$

As in the case of a planar interface, the boundary conditions at the nonplanar interface imply continuity of the electric $(\mathbf{E}(x,z) = (0, E(x,z), 0))$ and magnetic $(\mathbf{H}(x,z) = (H_x(x,z), 0, H_z(x,z)))$ field components that are tangent to the nonplanar interface.

The requirement of continuity of the tangent electric-field component $E(x,z)$ (the y component for the case of TE wave under consideration) has the form $E_1(x, f(x)) = E_2(x, f(x))$, where $E_{1(2)}(x,z)$ is the electric field in the first (second) medium (Fig. 1). If $f(x)$ is small in the sense of conditions (1), the aforementioned requirement can be written as

$$E_1(x,0) + \frac{\partial E_1(x,0)}{\partial z} f(x) = E_2(x,0) + \frac{\partial E_2(x,0)}{\partial z} f(x). \tag{4}$$

To obtain the condition corresponding to the continuity of the tangent magnetic-field components at the 1–2 interface, we introduce a unit vector of the tangent to the nonplanar interface between these media:

$$\begin{pmatrix} e_x(x) \\ e_z(x) \end{pmatrix} = \begin{pmatrix} \cos \arctan f(x) \\ \sin \arctan f(x) \end{pmatrix} \approx \begin{pmatrix} 1 \\ f(x) \end{pmatrix} + O(f^2(x)). \quad (5)$$

Here, we apply an assumption that $f'(x) \ll 1$. Using the introduced vector (5), one can write magnetic-field component $H_{\|}$ that is tangent to the 1–2 interface as $H_{\|} = H_x e_x(x) + H_z e_z(x)$. Let $\mathbf{H}_1(x,z)$ ($\mathbf{H}_2(x,z)$) be the magnetic field in medium 1 (2). Then the condition of continuity of the magnetic-field component tangent to the $z = f(x)$ interface has the form

$$\begin{gathered} H_{1x}(x, f(x)) e_x(x) + H_{1z}(x, f(x)) e_z(x) \\ = H_{2x}(x, f(x)) e_x(x) + H_{2z}(x, f(x)) e_z(x). \end{gathered}$$

Having written this relation accurate to the first-order terms with respect to $f(x)$ and $f'(x)$, we obtain

$$\begin{gathered} H_{1x}(x,0) + \frac{\partial H_{1x}(x,0)}{\partial z} f(x) + H_{1z}(x,0) f'(x) \\ = H_{2x}(x,0) + \frac{\partial H_{2x}(x,0)}{\partial z} f(x) + H_{2z}(x,0) f'(x). \end{gathered} \quad (6)$$

Using the Maxwell equation $\operatorname{curl}\mathbf{E} = -\imath k \mathbf{H}$, one can express the magnetic-field components $H_{x,z}$ entering (6) in terms of the y component of electric field E (in the case of TE waves under consideration, it is the only nonzero electric-field component). Thus, we have

$$H_{xi} = \frac{\imath}{k} \frac{\partial E_i}{\partial z}, \quad H_{zi} = -\frac{\imath}{k} \frac{\partial E_i}{\partial x}, \quad i = 1, 2. \quad (7)$$

Here, i is the number of the medium in which the field is considered. Substituting these expressions into (6), we obtain the following boundary condition for the TE electric field at the nonplanar 1–2 interface:

$$\begin{gathered} \frac{\partial E_1(x,0)}{\partial z} + \frac{\partial^2 E_1(x,0)}{\partial z^2} f(x) - \frac{\partial E_1(x,0)}{\partial x} f'(x) \\ = \frac{\partial E_2(x,0)}{\partial z} + \frac{\partial^2 E_2(x,0)}{\partial z^2} f(x) - \frac{\partial E_2(x,0)}{\partial x} f'(x). \end{gathered} \quad (8)$$

Conditions (4) and (8) in combination with the boundary conditions at the planar 2–3 interface, which imply the requirements of continuity of the field and its first derivative over z,

$$E_2(x,-L) = E_3(x,-L), \quad \frac{\partial E_2(x,-L)}{\partial z} = \frac{\partial E_3(x,-L)}{\partial z}, \quad (9)$$

allow us to write a closed system of equations for determining the desired scattered field. To this end, we present the fields in all structure portions as expansions in plane waves. The field expansion in the upper half-space can be written as

$$E_1(x,z) = A e^{-\imath k n_1 [\sin\theta x - \cos\theta z]} + \int dq R_q e^{\imath [qx - \sqrt{k^2 n_1^2 - q^2} z]}, \quad z > f(x). \quad (10)$$

Here, the first term is the wave incident on the structure and the second term corresponds to the ascending waves scattered into the upper half-space.

The field in medium 2 (we refer to this part of the structure as a "waveguide layer") contains both ascending and descending waves:

$$\begin{gathered} E_2(x,z) \\ = \int dq \left[D_q e^{\imath \left[qx + \sqrt{k^2 n_2^2 - q^2} z \right]} + U_q e^{\imath \left[qx - \sqrt{k^2 n_2^2 - q^2} z \right]} \right], \\ -L < z < f(x). \end{gathered} \quad (11)$$

Field $E_3(x,z)$ in the substrate portion between the waveguide layer and the QW (spaced from the waveguide layer by distance Z) may also contain ascending and descending harmonics:

$$E_3(x,z) = \int dq \left\{ T_q e^{\imath [qx + \sqrt{k^2 n_3^2 - q^2} z]} + T_q^+ e^{\imath [qx - \sqrt{k^2 n_3^2 - q^2} z]} \right\}, \quad -Z < z < -L. \quad (12)$$

The amplitudes of the descending (T_q) and ascending (T_q^+) waves are related via the QW reflectance $æ_q$ ($T_q^+ = æ_q T_q$), which is assumed to be known. For further convenience, we introduce the following parameters:

$$p_{1x} \equiv k n_1 \sin\theta, \quad p_{iz} \equiv \sqrt{k^2 n_i^2 - p_{1x}^2}, \quad i = 1, 2, 3. \quad (13)$$

Then, based on the requirement of continuity of the field and its first derivative at the 2–3 interface and the boundary conditions (4) and (8), one can write the following system of equations for amplitudes D_q, U_q, T_q, and R_q of the waves in the layers:

$$\begin{gathered} D_q e^{-\imath L \sqrt{k^2 n_2^2 - q^2}} + U_q e^{\imath L \sqrt{k^2 n_2^2 - q^2}} \\ = T_q e^{-\imath L \sqrt{k^2 n_3^2 - q^2}} \left[1 + æ_q e^{2\imath L \sqrt{k^2 n_3^2 - q^2}} \right], \\ D_q e^{-\imath L \sqrt{k^2 n_2^2 - q^2}} - U_q e^{\imath L \sqrt{k^2 n_2^2 - q^2}} \\ = \left(\frac{n_2}{n_3} \right)^{\alpha} \sqrt{\frac{k^2 n_3^2 - q^2}{k^2 n_2^2 - q^2}} T_q e^{-\imath L \sqrt{k^2 n_3^2 - q^2}} \left[1 - æ_q e^{2\imath L \sqrt{k^2 n_3^2 - q^2}} \right], \end{gathered} \quad (14)$$

$$\begin{gathered} R_q - D_q - U_q - \imath \int ds \Big[R_s \sqrt{k^2 n_1^2 - s^2} \\ + (D_s - U_s) \sqrt{k^2 n_2^2 - s^2} \Big] F_{q-s} + \imath A p_{1z} F_{q+p_{1x}} \\ = -A \delta(q + p_{1x}), \end{gathered} \quad (15)$$

$$
\begin{gathered}
A[\imath p_{1z}\delta(q+p_{1x}) - p_{1z}^2 F_{q+p_{1x}} - p_{1x}(q+p_{1x})F_{q+p_{1x}}] \\
-\imath R_q\sqrt{k^2n_1^2-q^2} + \int ds F_{q-s}R_s[qs-k^2n_1^2] \\
= \left(\frac{n_1}{n_2}\right)^{\alpha}\left\{\imath\sqrt{k^2n_2^2-q^2}[D_q-U_q] \right. \\
\left. + \int ds[D_s+U_s]F_{q-s}[qs-k^2n_2^2]\right\}.
\end{gathered} \tag{16}
$$

Here, the first pair of equations corresponds to the continuity of the field and its derivative with respect to z at the 2–3 interface, while the second pair corresponds to conditions (4) and (8). For the TE wave under consideration, $\alpha = 0$. System of equations (14)–(16) describes the case of a TH wave as well, where the only nonzero magnetic-field component is the y component (H_y). To this end, we assume that $\alpha = 2$ and make replacement $E \to H_y$.

The derived system of equations (14)–(16) completely describes the TR waveguide–coupling diffraction grating–QW layered structure. Moreover, this system can be used to analyze layered structures obtained from that the above-considered one using a particular simplification. For example, the experiments on light injection into a TR waveguide (see below) were performed on a structure without QW. The equations describing this structure can be devoted from system (14)–(16) on the assumption that the QW reflectance is zero: $æ_q = 0$. To estimate the depth of the coupling diffraction grating, the scattering problem must be solved for one nonplanar 1–2 interface (without a waveguide). The corresponding equations, which make it possible to find diffracted field R_q (D_q) scattered into the upper (lower) half-space, can be derived from system (14)–(16) if Eqs. (14) describing the reflection at the 2–3 interface are rejected and amplitudes U_q and T_q in the remaining equations (15) and (16) are assumed to be zero. In the next section, we present the solution to system (14)–(16) for a layered structure of TR waveguide with a coupling diffraction grating, obtained within the single-scattering approximation (Born approximation).

3. SINGLE-SCATTERING APPROXIMATION (CASE OF TE WAVE)

In this section, we consider the case of TE-polarized incident field. The single-scattering approximation is a widespread method for analyzing various problems of scattering in quantum mechanics, optics, acoustics, etc. For the above-stated problem, this approximation corresponds to the solution of system of equations (14)–(16) accurate to the first-order terms with respect to F_q, which are assumed to be small. As a result, the solution to this system will be sought in the form

$$
\begin{aligned}
D_q &= D^0\delta(q+p_{1x}) + D_q^1 + O(F_q^2), \\
U_q &= U^0\delta(q+p_{1x}) + U_q^1 + O(F_q^2), \\
T_q &= T^0\delta(q+p_{1x}) + T_q^1 + O(F_q^2), \\
R_q &= R^0\delta(q+p_{1x}) + R_q^1 + O(F_q^2).
\end{aligned} \tag{17}
$$

The first terms in these expressions have zero order with respect to F_q, and their dependences on the x projection of the wave vector (q) of the waves under consideration have the form $\delta(q+p_{1x})$, because these terms describe in fact the refraction and reflection of the incident wave by a structure with planar boundaries.

The next terms (D_q^1, U_q^1, T_q^1, and R_q^1) are of the first order with respect to F_q, and the derivation of the explicit expressions for them correspond to the determination of the scattered field within the single-scattering approximation. To solve this problem, the zero-order solution (R^0, D^0, U^0, and T^0) must be found. The equations for these parameters can be obtained by substituting expansion (17) into system (14)–(16) with all terms of orders higher than zero with respect to F_q disregarded. For further convenience, we introduce a matrix $M(q)$ in the form

$$
M(q) \equiv \begin{pmatrix} 0 & e^{-\imath g_2 L} & e^{\imath g_2 L} & -e^{-\imath g_3 L}æ_q^+ \\ 0 & g_2 e^{-\imath g_2 L} & -g_2 e^{\imath g_2 L} & -g_3 e^{-\imath g_3 L}æ_q^- \\ 1 & -1 & -1 & 0 \\ g_1 & g_2 & -g_2 & 0 \end{pmatrix}, \tag{18}
$$
$$
g_i \equiv \sqrt{k^2n_i^2-q^2}, \quad i = 1,2,3,
$$

and two column vectors $H(q)$ and $G(q)$, which are the solutions to the equations

$$
M(q)H(q) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad M(q)G(q) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}. \tag{19}
$$

Here, the designations $æ_q^{\pm} = 1 \pm æ_q e^{2\imath L\sqrt{k^2n_3^2-q^2}}$ are introduced. Then, as the direct calculation shows, the equation for the zero-order column vector (which is denoted as X) can be written as

$$
M(-p_{1x})X = A\begin{pmatrix} 0 \\ 0 \\ -1 \\ p_{1z} \end{pmatrix}, \quad X \equiv \begin{pmatrix} R^0 \\ D^0 \\ U^0 \\ T^0 \end{pmatrix}, \tag{20}
$$

and the desired vector X (solution to Eq. (20)) can be expressed in terms of vectors (19) as follows:

$$X = Ap_{1z}H(p_{1x}) - AG(p_{1x}). \tag{21}$$

Below, we present the expressions for vectors $H(q)$ and $G(q)$, which can be obtained by explicit solution of Eqs. (19):

$$H(q) = \begin{pmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{pmatrix} = \frac{1}{\Delta(q)} \begin{pmatrix} g_3æ_q^- + g_2æ_q^+ + (g_2æ_q^+ - g_3æ_q^-)e^{-2\imath g_2 L} \\ g_3æ_q^- + g_2æ_q^+ \\ (g_2æ_q^+ - g_3æ_q^-)e^{-2\imath g_2 L} \\ 2g_2 e^{\imath[g_3 - g_2]L} \end{pmatrix} \equiv \frac{1}{\Delta(q)} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{pmatrix}, \tag{22}$$

$$G(q) = \begin{pmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \end{pmatrix} = -\frac{1}{\Delta(q)} \begin{pmatrix} g_2[e^{-2\imath g_2 L}(g_2æ_q^+ - g_3æ_q^-) - g_2æ_q^+ - g_3æ_q^-] \\ g_1(g_2æ_q^+ + g_3æ_q^-) \\ g_1(g_2æ_q^+ - g_3æ_q^-)e^{-2\imath g_2 L} \\ 2g_1 g_2 e^{\imath(g_3 - g_2)L} \end{pmatrix} \equiv \frac{1}{\Delta(q)} \begin{pmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \end{pmatrix}. \tag{23}$$

Here, function $\Delta(q)$ is determined as

$$\begin{aligned} \Delta(q) &= (g_2 + g_1)(g_3æ_q^- + g_2æ_q^+) \\ &+ e^{-2\imath g_2 L}(g_2æ_q^+ - g_3æ_q^-)(g_1 - g_2), \\ g_i &= \sqrt{k^2 n_i^2 - q^2}. \end{aligned} \tag{24}$$

It will be shown below that only the factor $1/\Delta(q)$ is responsible for the singular behavior of vectors G and H with respect to q; therefore, it is reasonable to introduce vectors h and g, which regularly depend on q, into formulas (22) and (23).

Applying formulas (22) and (23), we can write the solution to Eq. (20) in the zero approximation:

$$\begin{pmatrix} R^0 \\ D^0 \\ U^0 \\ T^0 \end{pmatrix} = \frac{A}{\Delta(q)} \left. \begin{pmatrix} [p_{3z}æ_q^- + p_{2z}æ_q^+][p_{1z} - p_{2z}] + [p_{1z} + p_{2z}](p_{2z}æ_q^+ - p_{3z}æ_q^-)e^{-2\imath p_{2z} L} \\ 2p_{1z}[p_{3z}æ_q^- + p_{2z}æ_q^+] \\ 2p_{1z}(p_{2z}ae_q^+ - p_{3z}æ_q^-)e^{-2\imath p_{2z} L} \\ 4p_{1z}p_{2z}e^{\imath[p_{3z} - p_{2z}]L} \end{pmatrix} \right|_{q = -p_{1x}}. \tag{25}$$

Now, the first approximation with respect to F_q can be calculated. Substituting representation (17) into Eqs. (14)–(16) and equating the first-order terms with respect to F_q, we find that the column vector of first-order corrections with respect to F_q should satisfy the equation

$$M(q)\begin{pmatrix} R_q^1 \\ D_q^1 \\ U_q^1 \\ T_q^1 \end{pmatrix} = \imath F_{q+p_{1x}} \begin{pmatrix} 0 \\ 0 \\ (R^0 - A)p_{1z} + (D^0 - U^0)p_{2z} \\ (R^0 + A)[k^2 n_1^2 + qp_{1x}] - (D^0 + U^0)[k^2 n_2^2 + qp_{1x}] \end{pmatrix}, \tag{26}$$

where the q-dependent matrix $M(q)$ is determined by formula (18). The solution to this system of equations can be expressed in terms of the above-introduced vectors $H(q)$ (22) and $G(q)$ (23):

$$\begin{aligned} \begin{pmatrix} R_q^1 \\ D_q^1 \\ U_q^1 \\ T_q^1 \end{pmatrix} &= \imath F_{q+p_{1x}} H(q)[(R^0 + A)[k^2 n_1^2 + qp_{1x}] - (D^0 + U^0)[k^2 n_2^2 + qp_{1x}]] \\ &+ \imath F_{q+p_{1x}} G(q)[(R^0 - A)p_{1z} + (D^0 - U^0)p_{2z}]. \end{aligned} \tag{27}$$

Expression (27) makes it possible to find the scattered fields in all regions of the layered structure under study within the single-scattering approximation using formulas (17), (12), (11), and (10) and, thus, completely solve the problem stated in the beginning of Section 2. As follows from these expressions, the scattered field is a sum of plane-wave components (harmonics). Below, we will consider only the scattered-field components corresponding to the waveguide modes of layer 2. These components will be calculated in the next section.

4. WAVEGUIDE MODE

It follows from Eq. (26) that, in the general case, the amplitudes of harmonics R_q^1, D_q^1, U_q^1, and T_q^1 of the scattered fields in the layers of the structure under study turn to zero at $A = 0$. Exceptions are harmonics with wave numbers $q = q_m$ ($m = 1, 2, \dots, N$), for which the determinant of the matrix $M(q)$ turns to zero and Eq. (26) may have a nonzero solution at the zero right-hand side. Specifically these harmonics correspond to the waveguide mode of the electromagnetic field in the layered structure under consideration, and it can be shown that the condition $\det M(q) = 0$ is equivalent to the condition $\Delta(q) = 0$, where $\Delta(q)$ is determined by formula (24). Roots $q = q_m$ ($m = 1, 2, \dots, N$) of the equation $\Delta(q) = 0$ depend on the optical-field frequency ω: $q_m = q_m(\omega)$. At $æ_q = 0$ (i.e., in the absence of QW or other objects near the waveguide), the functions $q_m(\omega)$ ($m = 1, 2, \dots, N$) are real and correspond to the dispersion relations of modes of the isolated TR waveguide, the number N of which depends on the thickness L of waveguide layer 2 (Fig. 1).

It follows from formula (27) and expressions (22) and (23) for H and G that the first-correction vector $(R_q^1, D_q^1, U_q^1, T_q^1)$ has poles at $q = q_m$ ($m = 1, 2, \dots, N$) and, therefore, at $q \approx q_m$ its components can be written as

$$R_q^1 \approx \frac{R_m}{q - q_m}, \quad U_q^1 \approx \frac{U_m}{q - q_m}, \qquad D_q^1 \approx \frac{D_m}{q - q_m}, \quad T_q^1 \approx \frac{T_m}{q - q_m}. \tag{28}$$

The expressions for pole amplitudes R_m, D_m, U_m, and T_m will be given below. A substitution of (28) into (26) and the limiting transition $q \to q_m$ show that the column vector composed of the pole amplitudes satisfies the homogeneous equation

$$M(q_m)\begin{pmatrix} R_m \\ D_m \\ U_m \\ T_m \end{pmatrix} = 0. \tag{29}$$

Here, we use the fact that the right-hand side of (26) remains finite at the limiting transition $q \to q_m$. Let us show that specifically the singular parts of (28) allow us to calculate the desired amplitudes of the waveguide modes in layer 2.

Let the coupling diffraction grating (Fig. 1) be located near the coordinate origin $x = 0$. The waveguide-mode excitation manifests itself in the fact that the electromagnetic field in waveguide layer 2 does not decay at $|x| \to \infty$. Certainly, we deal with the field part that arose due to the incident-wave scattering from the coupling diffraction grating and has the order of smallness $\sim F_q$. Using formulas (11) and (17), we find that this part (we retain the designation $E_2(x, z)$ for it) is determined by the relation

$$E_2(x,z) = \int dq e^{\imath qx}\left[D_q^1 e^{\imath\sqrt{k^2 n_2^2 - q^2}z} + U_q^1 e^{-\imath\sqrt{k^2 n_2^2 - q^2}z}\right] + \sum_{m=0}^{N} C_m e^{\imath q_m x}\left[D_m e^{\imath\sqrt{k^2 n_2^2 - q_m^2}z} + U_m e^{-\imath\sqrt{k^2 n_2^2 - q_m^2}z}\right], \tag{30}$$
$$-L < z < f(x).$$

This expression differs from (11) by the presence of the second term, which is a sum of the solutions to homogeneous Eq. (29) taken with weighting factors C_m. These factors will be determined below based on additional conditions, which the desired waveguide mode of electromagnetic field in the structure (Fig. 1) should satisfy. From the physical point of view, the second term in (30) is the electromagnetic field of waveguide modes, which may occur in the structure even when the incident wave is absent (i.e., at $A = 0$).

The integral term in (30) does not tend to zero at large x only when the expression in the square brackets has singularities in the range of integration. Otherwise, this integral term tends to zero at $x \to \infty$ due to the rapidly oscillating factor $e^{\imath qx}$. As was shown above, the singularities of functions D_q^1 and U_q^1 are poles; therefore, the following formula can be used to calculate the integral in (30) at $|x| \to \infty$:

$$\int_a^b \frac{\Phi(q)e^{\imath qx}}{q - q_m} dq \approx -\imath\,\mathrm{sgn}(x)\pi\Phi(q_m)e^{\imath q_m x}, \tag{31}$$
$$q_m \in [a, b],$$

which is valid for any function $\Phi(q)$ that is regular at $q \in [a, b]$. Using (31) and (28), we derive the following expression for the field in waveguide layer 2 at long distances:

$$E_2(x,z) = -\imath\,\mathrm{sgn}(x)\pi\sum_{m=1}^{N} e^{\imath q_m x}\left[D_m e^{\imath\sqrt{k^2 n_2^2 - q_m^2}z} + U_m e^{-\imath\sqrt{k^2 n_2^2 - q_m^2}z}\right] \tag{32}$$
$$+ \sum_{m=0}^{N} C_m e^{\imath q_m x}\left[D_m e^{\imath\sqrt{k^2 n_2^2 - q_m^2}z} + U_m e^{-\imath\sqrt{k^2 n_2^2 - q_m^2}z}\right]$$

at $|x| \to \infty$.

Constants C_m entering this formula can now be found as follows. For clarity, q_m is assumed to be negative for the mth mode of our waveguide. Consequently, the corresponding wave propagates along the x axis in the positive direction. We assume that this mode can be excited in the structure shown in Fig. 1 only due to the incident-wave scattering from the coupling diffraction grating located near the coordinate origin. Therefore, it should be required that the field of this mode is nonzero only at positive x values and turns to zero at $x = -\infty$. This requirement can be satisfied assuming that $C_m = -\imath\pi$. In this case, the field in the waveguide gap at positive x is doubled. Based on this reasoning, we can conclude that the field in the waveguide gap at large distances ($|x| \to \infty$) from the coupling diffraction grating is generally determined by the formula

$$E_2(x,z) = -2\imath\pi\sum_{m=1}^{N} e^{\imath q_m x}\Big[D_m e^{\imath\sqrt{k^2n_2^2-q_m^2}z} + U_m e^{-\imath\sqrt{k^2n_2^2-q_m^2}z}\Big]\Theta(-q_m x). \tag{33}$$

Using expressions (27), (23), and (22), we can easily obtain the following relations for the pole amplitudes entering the above formula:

$$D_m = \frac{\imath F_{q_m+p_{1x}}}{\Delta'_m}\Big(æ^+_{q_m}\sqrt{k^2n_2^2-q_m^2} - \imath æ^-_{q_m}\sqrt{q_m^2-k^2n_3^2}\Big)J_m, \tag{34}$$

$$U_m = \frac{\imath F_{q_m+p_{1x}}}{\Delta'_m}e^{-2\imath\sqrt{k^2n_2^2-q_m^2}L} \times \Big(æ^+_{q_m}\sqrt{k^2n_2^2-q_m^2} + \imath æ^-_{q_m}\sqrt{q_m^2-k^2n_3^2}\Big)J_m, \tag{35}$$

where

$$\Delta'_m \equiv \left.\frac{\partial\Delta(-\imath\sqrt{q^2-k^2n_1^2},\sqrt{k^2n_2^2-q^2},-\imath\sqrt{q^2-k^2n_3^2})}{\partial q}\right|_{q=q_m},$$

$$J_m \equiv (R^0+A)[k^2n_1^2+q_m p_{1x}] - (D^0+U^0)[k^2n_2^2+q_m p_{1x}] + \imath\sqrt{q_m^2-k^2n_1^2}[(R^0-A)p_{1z}+(D^0-U^0)p_{2z}].$$

Here, the first and third arguments of the function $\Delta(g_1, g_2, g_3)$ correspond to evanescent waves in media 1 and 3. Thus, with the q_m values known, formulas (33), (34), and (35) make it possible to find the field in the waveguide gap that is excited by the incident wave via the coupling grating.

To determine wave numbers q_m of the waveguide modes, one should solve the equation $\Delta(q) = 0$, which is analyzed below. Taking into account that

$$æ^\pm_q \equiv 1 \pm æ_q e^{2\imath L\sqrt{k^2n_3^2-q^2}} = 1 \pm æ_q e^{2\imath L g_3},$$

we find (using (24)) that the equation $\Delta(q) = 0$ can be expanded as

$$\frac{(g_2+g_1)[g_2+g_3+æ_q(g_2-g_3)e^{2\imath Lg_3}]}{(g_2-g_1)[g_2-g_3+æ_q(g_2+g_3)e^{2\imath Lg_3}]} = e^{-2\imath Lg_2}, \quad g_i = \sqrt{k^2n_i^2-q^2}, \quad i = 1, 2, 3. \tag{36}$$

The roots q_m of this equation depend on frequency ω in terms of parameters g_i and $æ_q$.

QW near the Waveguide

Let us consider the case in which the layered system under study is a combination of a TR waveguide and a thin QW with the optical (exciton) resonance at frequency ω_0 spaced by a distance $Z > L$ from the $z = 0$ plane (Fig. 1). A similar analysis was performed in [13]. The QW susceptibility is assumed to be presentable in the form of a simple pole $\sim 1/[\omega_0 - \omega]$. Having denoted the radiative width (see, for example, [14]) of the reflection spectrum of this well as Γ_0, one can obtain the following expressions for reflectance $æ_q$ and parameters $æ^\pm_q$:

$$æ_q - \frac{e^{-2Z\sqrt{q^2-k^2n_3^2}}}{[\omega_0-\omega]\sqrt{(q/k)^2-n_3^2}/\Gamma_0+1}, \quad æ^\pm_q = 1 \mp \frac{e^{-\xi\sqrt{n^2-n_3^2}}}{\delta\nu\sqrt{n^2-n_3^2}+1}, \tag{37}$$

where the following designations are introduced:

$$\delta\nu \equiv \frac{\omega_0-\omega}{\Gamma_0}, \quad \xi \equiv \frac{4\pi(Z-L)}{\lambda}, \quad l \equiv \frac{2\pi L}{\lambda}, \quad n \equiv \frac{q}{k}, \quad \lambda \equiv \frac{2\pi}{k}. \tag{38}$$

Expressions (37) suggest that $q > kn_3$; therefore, $æ_q$ and $æ^\pm_q$ are real values. Based on these relations, Eq. (36) is reduced to the form

$$\arctan\sqrt{\frac{n^2-n_1^2}{n_2^2-n^2}} + \arctan\left\{\sqrt{\frac{n^2-n_3^2}{n_2^2-n^2}}\frac{1+\delta\nu\sqrt{n^2-n_3^2}+e^{-\xi\sqrt{n^2-n_3^2}}}{1+\delta\nu\sqrt{n^2-n_3^2}-e^{-\xi\sqrt{n^2-n_3^2}}}\right\} = l\sqrt{n_2^2-n^2}+\pi m. \tag{39}$$

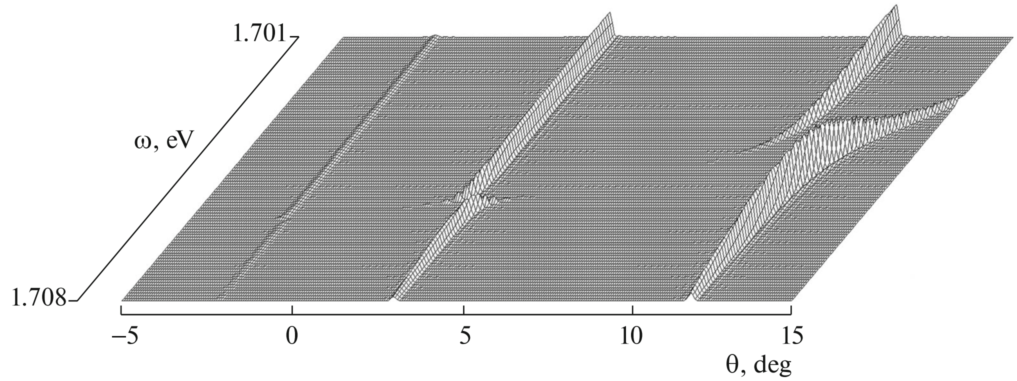

Fig. 2. Polariton mode in the TR waveguide–QW structure (Fig. 1). Angle of incidence θ of the excitation wave on the coupling diffraction grating is plotted on the horizontal axis, $\sum_m |U_m|^2$ is plotted on the vertical axis, and excitation-wave frequency ω is given in the third direction. The structural parameters are as follows: waveguide-layer thickness $L = 1$ μm and refractive indices of the layers $n_1 = 1$, $n_2 = 3.6$, and $n_3 = 3.2$. The coupling diffraction grating is described by formula (3) with the following parameters: spatial period $2\pi/s = 300$ nm and length $2B = 300$ μm. The QW parameters are as follows: radiative width $\Gamma_0 = 0.0005$ eV, resonant frequency $\omega_0 = 1.17$ eV, and distance to the waveguide $Z - L = 300$ nm.

Equation (39) has roots $n = n_m = q_m/k$ only at some integers m, which are numbers of modes. A graphical analysis of Eq. (39) shows that, at a specified m, the roots of this equation can be in the range $n \in [n_3, n_2]$ (recall that we assume that $n_1 < n_3 < n_2$). A transition to the purely waveguide structure (i.e., QW is "switched off") corresponds to any of the following passages to the limit: $\Gamma_0 \to 0$ or $Z \to \infty$. In both these limiting cases, Eq. (39) is transformed into the known dispersion equation for a TE wave in a planar waveguide [7, 8], which can have no more than one root at specified frequency ω and mode number m. The numerical solution of Eq. (39) shows that the presence of a QW with optical resonance near the waveguide may give rise to the second root of this equation, which corresponds to the presence of two polariton modes with the same frequency ω. This fact distinguishes the structure under consideration from that described in [6], where the QW was located inside the waveguide. If such a structure (waveguide + QW) is excited using a coupling diffraction grating, the electromagnetic field and its intensity in the waveguide layer can be calculated from formulas (25), (33), (34), (35), and (37). An example of this calculation is shown in Fig. 2. The structural parameters are in the figure caption. In this case, there is an interaction between two electromagnetic field modes: polariton wave localized near the QW and waveguide mode enclosed in the waveguide gap. The dispersion relation for the QW polariton can be obtained as the condition of infinity of reflectance (37) (i.e., $æ_q = \infty$):

$$\omega = \omega_0 + \frac{\Gamma_0}{\sqrt{n^2 - n_3^2}}, \quad n = \frac{q}{k}. \tag{40}$$

Planar Interface

Let us now consider the case in which the reflecting object under the waveguide layer is a planar interface at $z = -Z$, $Z > 0$, beyond which an infinite medium with real refractive index n_4 begins. This case corresponds, for example, to the situation, where a waveguide structure is grown on a substrate (light is inevitably reflected from the interface between the substrate and air). Here, one can show [15] that

$$æ_q = e^{-2ig_3Z} \frac{g_3 - g_4}{g_3 + g_4}, \tag{41}$$

where $g_4 = \sqrt{k^2 n_4^2 - q^2}$. If the substrate thickness is much larger than the damping length of the evanescent waves under total internal reflection of the ith mode in the waveguide layer (i.e., $|g_3(q_i)Z| \gg 1$ and parameter $g_3(q_i)$ is purely imaginary), the waveguide mode leaking through the substrate can be disregarded (i.e., $æ_{q_i}$ can be considered zero). However, parameters $æ_{-p_{1x}}$ entering expression (25) for the zero approximation are not small and should be taken into account (they describe the interference phenomena related to the reflection from the substrate–environment interface). For example, the field strength near the coupling diffraction grating may change significantly with a temperature-induced change in the optical substrate thickness, which, in turn, significantly changes the strength in the waveguide gap. It is important that one can change the degree of the coupling between the waveguide and excitation beam from strong (the grating is in a field antinode) to weak (the grating is in a field node) by changing the substrate

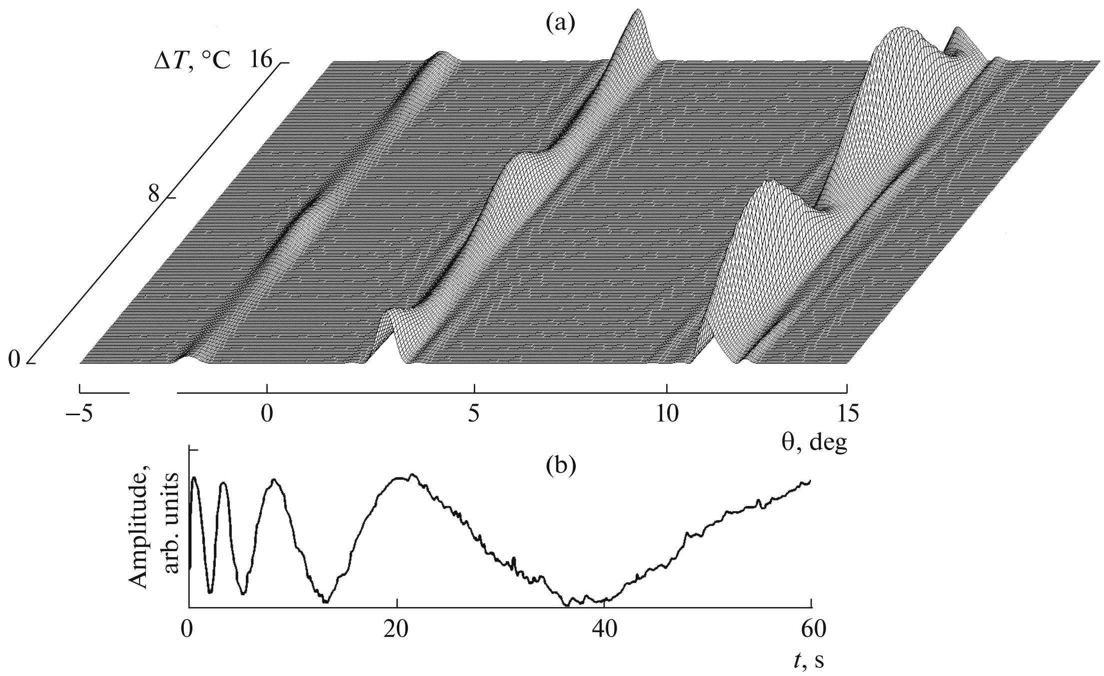

Fig. 3. (a) Change in intensity $\sum_m |U_m|^2$ of the waveguide modes (m = 0, 1, and 2) with a temperature-induced change in the substrate thickness. The angle of incidence of the excitation beam on the structure is plotted on the horizontal axis. Change in temperature ΔT is plotted inward. The temperature dependence of the substrate refractive index was chosen in the form $n_3 = 3.2 + \beta\Delta T$, $\beta = 2\times10^{-4}$, $\Delta T \in [0,16]$°C. The length of the coupling diffraction grating is $2B = 100$ μm, substrate thickness $Z = 400$ μm, and the other parameters are the same as in Fig. 2. (b) Experimental time dependence of the fundamental-mode amplitude upon cooling the waveguide structure from ~60 to 20°C.

thickness. Figure 3a shows the temperature behavior of the angular dependence of the intensity in the waveguide gap of the structure shown in Fig. 1 with three modes (m = 0, 1, and 2) without QW. The structural parameters are as follows: $L = 1$ μm, $n_1 = 1$, $n_2 = 3.6$, $n_3 = 3.2 + \beta\Delta T$, and $\beta \approx 2\times10^{-4}$. The substrate thickness is $Z = 400$ μm. We assumed in the calculations that the optical path in the substrate changes due to the dependence of refractive index n_3 on change in temperature ΔT, and coefficient β was taken from [16]. The results of these calculations are in correspondence with Fig. 3b, which shows the time dependence of the fundamental-mode amplitude of the real structure with the aforementioned parameters, initially heated to ~60°C by a heat blower and cooled to 20°C. The experimental setup used for the observation will be described in the next section.

The above consideration concerns the case of the TE-polarized incident field. One can obtain relations for TH waves similar to (33)–(41) in a methodologically similar way by changing the boundary conditions.

5. EXPERIMENTS ON LIGHT INJECTION INTO A TR WAVEGUIDE USING A DIFFRACTION GRATING

To observe the light injection into the above-described waveguide structure, we used a system presented in a simplified form in Fig. 4. Light beam *1* with a diameter of ~3 mm generated by neodymium laser *2* (wavelength $\lambda = 1.06$ μm), was incident on the surface of waveguide structure *3*. The sample was mounted on special holder *4*, which performed vibrational and rotational motion around the O axis with a frequency of ~15 Hz. At some angle of incidence, the wave vector of the beam diffracted into the waveguide was equal to that of one of the waveguide modes. In this case, the light intensity from the waveguide end facing fiber bundle *5* sharply increased. This increase was detected by photodetector *6* at the other end of the fiber bundle and displayed on oscilloscope *7*.

A structure similar to that in Fig. 1 but without QW was fabricated for the experiments. It was a GaAs waveguide layer with refractive index $n_2 = 3.6$ and thickness $L = 1$ μm, enclosed in a layer of $Al_{0.3}Ga_{0.7}As$ solid solution (refractive index $n_3 = 3.2$) and facing air (refractive index $n_1 = 1$) from the other side. Two diffraction gratings with length $2B = 300$ μm and period

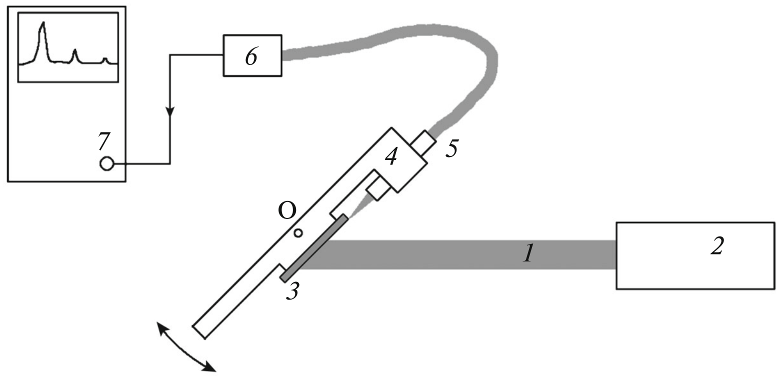

Fig. 4. Schematic of the setup for observing light injection into the TR waveguide using a diffraction grating.

$2\pi/s = 0.3$ μm, spaced by a gap of $\Delta x = 2$ mm, were formed on the waveguide-layer surface by plasma etching. Having denoted the amplitudes of these gratings as ε_1 and ε_2 (their values will be estimated below), we obtain the following expression for function F_q entering the above formulas:

$$F_q = \frac{1}{2\pi}\left[\frac{\sin(s-q)B}{s-q} + \frac{\sin(s+q)B}{s+q}\right](\varepsilon_1 + \varepsilon_2 e^{\imath q\Delta x}). \quad (42)$$

Figure 5a shows a typical angular dependence of the field intensity in the waveguide gap observed on the display of oscilloscope *7* under illumination of one of the gratings. One can see all three possible modes (upper curve); the relative excitation intensity for each mode is in qualitative correspondence with the results of the theoretical calculations (the lower curve, which is obtained using (33)–(35) at $æ_q = 0$ and function F_q in form (3)). We relate the presence of the additional peak (indicated by an arrow in the figure) to the spurious diffraction peak caused by the slight aperiodicity of the coupling diffraction grating.

We could illuminate both gratings simultaneously by scanning the incident beam over the sample surface. In this case, oscillations were observed in the output signal of photodetector *6* (Fig. 5b). This effect is described by formulas (33)–(35), in which $æ_q$ should be equated to zero and function F_q should be taken in form (42) at $\varepsilon_1/\varepsilon_2 = 0.3$ (Fig. 5c). In the calculations, we estimated the recorded field intensity in the waveguide as $\sum_m |U_m|^2$ (35).

Estimation of the Amplitude of the Coupling Diffraction Grating

The coupling diffraction gratings formed on the surface of the above-described waveguide structure had a period of ~300 nm. This period was chosen so as to provide the waveguide excitation by a neodymium laser beam ($\lambda = 1.06$ μm) at small angles of incidence (about ~±5°). The diffraction of the second-harmonic green beam of neodymium laser ($\lambda = 0.532$ μm) can easily be observed for this grating. The obtained equations describing the scattering from the layered structure with one nonplanar interface make it possible to derive formulas for the diffraction efficiency of the coupling diffraction grating; measurement of the diffraction efficiency allows one to experimentally estimate the diffraction-grating amplitude ε without electron microscopy. To obtain the corresponding equations, one should, first, assume that $U_q = 0$ in Eqs. (15) and (16) and find reflected field R_q within the single-scattering approximation and, second, determine the total reflected-field intensity by integrating over q. Omitting the details of this calculation, we present the final formula for the TE-polarized incident wave:

$$\frac{E_s}{E_i} = \frac{\sigma}{S}\frac{\varepsilon^2 k^2 n_1 \cos^2\theta [n_1^2 - n_2^2]^2}{[n_1\cos\theta + \sqrt{n_2^2 - n_1^2\sin^2\theta}]^2} \times \frac{\sqrt{n_1^2 - (\lambda/b \pm n_1\sin\theta)^2}}{[\sqrt{n_1^2 - (\lambda/b \pm n_1\sin\theta)^2} + \sqrt{n_2^2 - (\lambda/b \pm n_1\sin\theta)^2}]^2}. \quad (43)$$

Here, E_s/E_i is the ratio of the intensities of the diffracted and incident laser beams, σ/S is the ratio of the grating area to the incident-beam cross section, θ is the angle of incidence of the beam on the grating, ε is the grating amplitude, and $b = 2\pi/s$ is the grating period. The other designations were introduced above. The sign ± should be chosen so as to make the radicals real at a specified sign of the angle of incidence. Figure 6a shows the experimental and calculated (using formula (43)) dependences of the diffraction efficiency on angle of incidence θ, which varied from the grazing value (–90°) to approximately –50°, when the diffracted beam "lay" on the grating plane. Based on the measured absolute value of diffraction efficiency E_s/E_i, the grating amplitude was estimated as $\varepsilon \approx 114$ nm. This result is in qualitative agreement with the electron microscopy image of our structure (Fig. 6b).

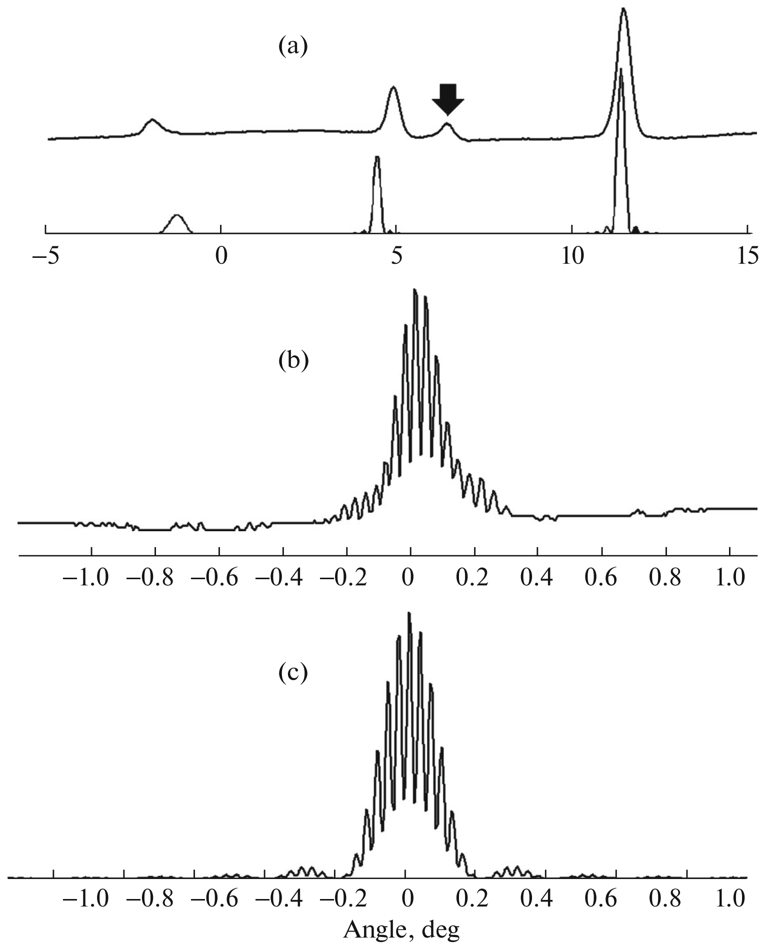

Fig. 5. (a) Angular dependence of the intensity in the waveguide gap: signal from photodetector *6* (Fig. 4) and $\sum_m |U_m|^2$ calculated from formula (35). (b, c) Waveguide excitation using two coupling diffraction gratings: (b) experimental data and (c) calculation result.

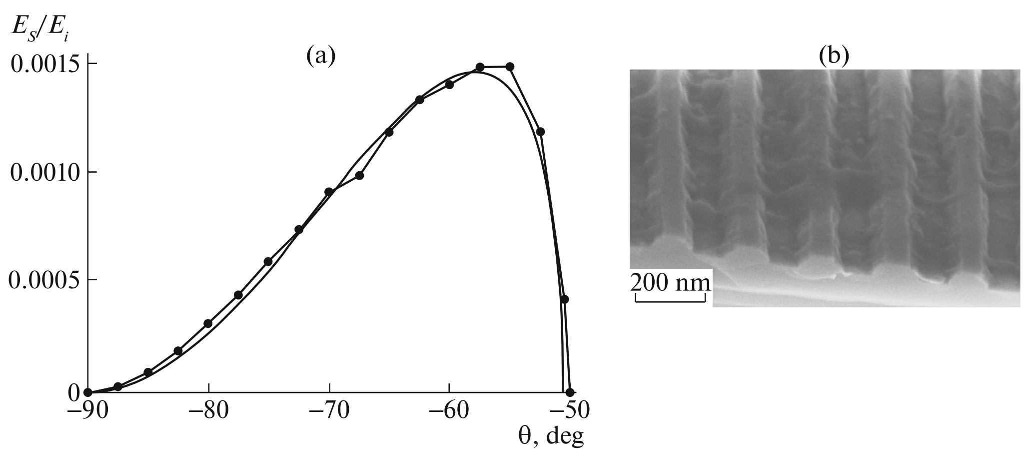

Fig. 6. (a) Angular dependence of the diffraction efficiency of the coupling diffraction grating in use, obtained at $\lambda = 0.532$ μm: (circles) experimental results and (solid line) calculation based on formula (43) at grating amplitude $\varepsilon = 114$ nm. (b) Micrograph of the coupling diffraction grating. To correctly estimate the grating amplitude, one must take into account that this projection is made at an angle of ~45°.

CONCLUSIONS

We developed a theory of scattering from a nonplanar interface in order to calculate the excitation of a planar TR waveguide using a diffraction grating. The equations for the scattered-field components were derived and solved within the single-scattering approximation, and the field in the waveguide gap was found. The proposed solution technique makes it possible to describe the polariton mode of the TR waveguide–QW combined structure. We reported the calculated and experimental results, which show that the presence of the reflecting boundary near the waveguide layer allows one to change the degree of coupling between the TR waveguide and incident light. Finally, using the developed technique of calculating the scattering from a nonplanar interface, we derived the formula for the diffraction efficiency, which made it possible to estimate the amplitude of the coupling diffraction grating on the samples studied.

Note that, although conditions (1) were poorly satisfied for the prepared gratings, the developed theoretical concepts could, nevertheless, be used to describe our experiments.

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