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SOLIDS AND LIQUIDS

Noise Spectroscopy of an Optical Microresonator

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Abstract—The noise spectrum is calculated for the intensity of light transmitted through an optical microresonator whose thickness experiences thermal oscillations. The noise spectrum reveals a maximum at the frequency of an acoustic mode localized in the optical microresonator and depends on the size of the illuminated region. The noise intensity estimates show that it can be detected by the modern noise spectroscopy technique.

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1. INTRODUCTION

The recent advent of fast digital electrical-signal spectrum analyzers has inspired renewed interest in optical noise spectroscopy, which involves a number of experimental techniques based on information contained in the noise spectrum on the intensity (or polarization) of light transmitted through a system under study.

In the first experiment of this kind [1], the electron paramagnetic resonance (EPR) of sodium atoms was observed in the noise spectrum of Faraday rotation. The high sensitivity of a polarimetric setup [2] ensured recording of the EPR spectrum (with a narrow resonance at ~ 1 MHz) by the successive spectrum accumulation method with scanning using the standard technique of low-frequency modulation of the resonance frequency combined with lock-in detection. The use of modern digital Fourier analyzers with parallel spectrum accumulation drastically reduced the spectrum accumulation time, made possible the observation of EPR spectra in a number of solid objects [3], and also extended the frequency region of the detected noise signal up to a few gigahertz. In addition, digital Fourier analyzers can be used to observe the response of a system to the noise modulation of the probe light [4–8]. Such measurements not only give information on the EPR spectrum of the system but also make it possible to estimate the parameters describing its interaction with light. All this suggests that noise spectroscopy, which was initiated in [1], will in the near feature undergo a rebirth based on new instrumental equipment with unique possibilities. For this reason, the search for new objects for noise spectroscopy (one of them is considered below) is of current interest.

The features of the frequency spectrum of the noise signal observed in the above-mentioned works were, as a rule, related to the magnetic structure of spin states of the system under study. At the same time, magnetic spin systems do not exhaust the variety of noise spectroscopy objects, and in this paper we present an example of a nonmagnetic optical system with noise that can be observed using the above-mentioned equipment. We describe the noise modulation of the intensity of quasi-monochromatic light transmitted through an optical microresonator. The modulation appears due to thermal fluctuations in the microresonator thickness (and, hence, its resonance frequency). According to calculations presented in our paper, this effect can be experimentally observed with the above-mentioned digital spectrum analyzer. In this paper, we only consider the possibility of observing this effect and do not discuss its informative properties. Note that, although effects similar to that described here have been studied earlier to estimate the sensitivity of laser interferometers in gravitational-wave detectors [9, 10], as far as we know, the noise of a thin interferometer in the context of noise spectroscopy has not been analyzed so far.

Let us explain in more detail the concept of the effect. Consider an optical resonator (a Fabry–Perot interferometer) consisting of two mirrors separated by distance L. If the reflection coefficient of the mirrors is close to unity, the frequency dependence of the transmission coefficient I/I_0 of such a resonator can be written in the form

$$\frac{I}{I_0} = \frac{\Delta^2}{\Delta^2 + (\omega - \omega_0)^2}. \quad (1)$$

Here, $I_0(I)$ is the intensity of a plane monochromatic wave with frequency ω at the input (output) of the interferometer and $\omega_0 = \pi c/L$ is the resonance frequency of the interferometer (c is the speed of light in the medium between mirrors). The width Δ of the transmission spectrum of the interferometer is determined by the reflection coefficient of mirrors, and quality factor Q for real microresonators, defined as

$Q \equiv \omega_0/\Delta$, can be on the order of 1000 or more. The change $L \longrightarrow L + \xi$ in the resonator thickness causes the change $\omega_0 \longrightarrow \omega_0 + \delta\omega_0$ in its resonance frequency, the relation $|\delta\omega_0/\omega_0| = |\xi/L|$ being fulfilled for $\xi/L \ll 1$. Taking this into account, we can easily see from (1) that the relative change $|\xi/L| \sim 1/Q$ in the resonator length leads to the change in its transmission coefficient on the order of unity and can easily be detected. In this case, the absolute value of the change in thickness ξ for typical values of the optical microresonator parameters $L = 0.25$ μm and $Q = 1000$ is on the order of the atom size ($\xi \sim 0.25$ nm). Such a high sensitivity of the optical resonator transmission to the change in its parameters is used in physical experiments (see, e.g., [11]) and suggests the possibility of observing fluctuations of the resonator transmission caused by thermal vibrations of the resonator thickness.[1] Thermal vibrations of the resonator thickness should give rise to the intensity noise of quasi-monochromatic light transmitted through the resonator, and to elucidate the possibility of observing this noise, its intensity should be compared with that of the shot noise of the light used in experiments. The corresponding calculations are presented in the next section.

2. MODEL CALCULATIONS

Calculation of the noise spectrum of a microresonator performed in this section is based on an extremely simplified model of the latter and, of course, does not pretend to detailed correspondence to a real optical system. The aim of this calculation is, first, to qualitatively describe the dependence of the noise spectrum on the experimental and microresonator parameters (the light beam intensity and diameter, the optical and acoustic Q factors of the microresonator, etc.) and, second, to quantitatively estimate the noise intensity in the spectral region where it considerably differs from zero.

Let us consider an optical resonator consisting of a medium layer L in thickness covered by thin mirrors. Let a monochromatic light beam with intensity I_0 and frequency ω be incident on the resonator. We will denote the area of a light spot on the resonator layer by D^2 and the resonator quality factor by $Q = \omega_0/\Delta$.[2] We will select the coordinated system so that the resonator plane would coincide with the xy plane. Then, the resonator thickness will be a function of x and y, which we represent as the sum of the constant average thickness L and small thermal fluctuations $\xi(t, x, y)$. We will assume that the resonance frequency of the resonator is determined by its thickness averaged over the light spot area D^2.[3] Then, the intensity fluctuation δI of light transmitted through the resonator can be written in the form

$$\delta I = G(\omega) \int_{D^2} dxdy\,\xi(t, x, y);$$

$$G(\omega) = \frac{I_0}{D^2} \frac{d}{d\omega_0}\left(\frac{\Delta^2}{\Delta^2 + (\omega - \omega_0)^2}\right) \frac{d\omega_0}{d\xi}\bigg|_{\xi = 0}, \tag{2}$$

$$\omega_0 = \frac{\pi c}{L + \xi}.$$

The maximum value of the factor $G(\omega)$

$$G(\omega_0 + \Delta) = I_0 Q/2LD^2$$

is obtained when $\omega = \omega_0 - \Delta$.

We are interested in the noise spectrum $S(\nu)$ of the light intensity transmitted through the resonator. The function $S(\nu)$ is related to the correlation function $\langle \delta I(0)\delta I(t)\rangle$ by the expression

$$S(\nu) = \int dt e^{i\nu t} \langle \delta I(0)\delta I(t)\rangle. \tag{3}$$

Using expression (2), we obtain for $\langle \delta I(0)\delta I(t)\rangle$ the expression

$$\langle \delta I(0)\delta I(t)\rangle = G^2(\omega) \int_{D^2}\int_{D^2} dxdydx'dy' \times \langle \xi(0, x, y)\xi(t, x', y')\rangle. \tag{4}$$

To calculate the correlation function $\langle \xi(0, x, y)\xi(t, x', y')\rangle$ entering (4), it is necessary to (i) specify a model of the motion of the resonator material, (ii) obtain the corresponding Hamiltonian H, and (iii) perform averaging in expression (4) with a thermodynamically equilibrium distribution function proportional to $\exp[-H/kT]$.

To describe the dynamics of the resonator material, we will use the simplest model according to which the motion of the resonator material represents weak sound waves. We will assume that the resonator is located in the region $z \in [0, L]$, $x \in [0, a]$, and $y \in [0, b]$. The optical transmission spectrum of the resonator depends on a change in its thickness in the z direction, and therefore we are interested only in the z

[1] Fluctuations in the resonance frequency of a microresonator are determined by a change in its optical thickness. Fluctuations in the geometrical thickness of the resonator lead to out-of-phase fluctuations in the material density of the resonator, resulting in fluctuations in the refractive index. This effect can reduce the sensitivity of the resonator frequency to variations in its thickness. However, the total compensation of the sensitivity is very improbable, and we will neglect this effect in our estimates presented below.

[2] The quality factor determined from the halfwidth of the transmission spectrum can depend on the transverse size D of the beam; however, in the case of normal incidence this dependence is weak.

[3] If the average thickness of the resonator within the light spot did not change, it is impossible to say in the linear in deformation approximation whether the resonator transmission increased or decreased. In this case, the change in transmission is caused by scattering, which we do not consider here.

projection of the displacement of the resonator material. According to the accepted model, this displacement represents the sound field $u(x, y, z)$ satisfying the wave equation

$$\frac{\partial^2 u}{\partial t^2} = v^2 \Delta u, \tag{5}$$

where v is the speed of sound in the resonator material. Energy related to sound field $u(x, y, z)$ is determined by the expression

$$E = \int dx dy dz \left\{ \frac{\rho \dot{u}^2}{2} + \frac{\gamma}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right]\right\}. \tag{6}$$

Here, ρ is the density of the resonator material and γ is a constant describing the density of the elastic deformation energy. The relation between this constant and speed of sound v will be presented below. Direct substitution shows that the expansion of the solution of Eq. (5) in normal modes (i.e., in solutions to Eq. (5) harmonically dependent on time) satisfying the boundary conditions

$$\left.\frac{\partial u}{\partial x}\right|_{x = 0, a} = 0, \quad \left.\frac{\partial u}{\partial y}\right|_{y = 0, b} = 0, \quad \left.\frac{\partial u}{\partial z}\right|_{z = 0, L} = 0, \tag{7}$$

corresponding to zero mechanical stress at the resonator boundaries has the form

$$u(t, x, y, z) = \sum_{pmn} u_{pmn} \cos\left(\frac{\pi p x}{a}\right) \times \cos\left(\frac{\pi m y}{b}\right) \cos\left(\frac{\pi n z}{L}\right), \tag{8}$$

where p, n, and $m > 0$ are integers; the degrees of freedom of the sound field (generalized coordinates) u_{pmn} satisfy the equations of motion

$$\ddot{u}_{pmn} = -\omega_{pmn}^2 u_{pmn}: \quad \omega_{pmn}^2 = v^2\left[\left(\frac{\pi p}{a}\right)^2 + \left(\frac{\pi m}{b}\right)^2 + \left(\frac{\pi n}{L}\right)^2\right]. \tag{9}$$

Using (8), we will express energy E (6) in terms of the degrees of freedom u_{pmn}:

$$E = \frac{V}{16}\sum_{pmn}\left[\rho \dot{u}_{pmn} + \gamma u_{pmn}^2\left[\left(\frac{\pi p}{a}\right)^2 + \left(\frac{\pi m}{b}\right)^2 + \left(\frac{\pi n}{L}\right)^2\right]\right] = \frac{V}{16}\sum_{pmn}\left[\rho \dot{u}_{pmn}^2 + \frac{\gamma \omega_{pmn}^2}{v^2} u_{pmn}^2\right]. \tag{10}$$

To obtain the Hamiltonian corresponding to energy (10), we should introduce generalized momenta p_{pmn} conjugated to generalized coordinates u_{pmn} so that equations of motion (9) will have the form of the Hamilton equations:

$$\frac{\partial H}{\partial u_{pmn}} = -\dot{p}_{pmn}, \quad \frac{\partial H}{\partial p_{pmn}} = \dot{u}_{pmn}. \tag{11}$$

By setting

$$p_{pmn} \equiv \frac{\dot{u}_{pmn} \rho V}{8}, \quad \gamma = \rho v^2,$$

and expressing energy (10) in terms of p_{pmn} and u_{pmn} as

$$H = \sum_{pmn}\left[\frac{4 p_{pmn}^2}{M} + \frac{M \omega_{pmn}^2 u_{pmn}}{16}\right], \tag{12}$$

where $M \equiv \rho V$ is the resonator mass, we can easily verify that equations of motion (9) are equivalent to (11). Thus, (12) is the required Hamiltonian.

Using Hamiltonian (12), we can write the distribution function $\sigma(\{p_{pmn}\}, \{u_{pmn}\})$ of generalized coordinates u_{pmn} and momenta p_{pmn} in the thermodynamically equilibrium state with inverse temperature $\beta = 1/kT$ in the form

$$\sigma(\{p_{pmn}\}, \{u_{pmn}\}) = Z^{-1} \exp[-\beta H(\{p_{pmn}\}, \{u_{pmn}\})], \tag{13}$$

where Z is the normalization constant. Using (8) to express the local change $\xi(t, x, y)$ in the resonator thickness in terms of degrees of freedom u_{pmn} as

$$\xi(t, x, y) = u(t, x, y, 0) - u(t, x, y, L) = 2\sum_{pmn} u_{pm, 2n-1} \cos\left(\frac{\pi p x}{a}\right) \cos\left(\frac{\pi m y}{b}\right), \tag{14}$$

where p, m, $n > 0$, we obtain the correlation function in (4) in the form

$$\langle \xi(0, x, y) \xi(t, x', y') \rangle = 4\sum_{pmn}\sum_{p'm'n'} \langle u_{pm, 2n-1}(0) u_{p'm', 2n'-1}(t) \rangle \cos\left(\frac{\pi p x}{a}\right) \times \cos\left(\frac{\pi m y}{b}\right) \cos\left(\frac{\pi p' x'}{a}\right) \cos\left(\frac{\pi m' y'}{b}\right). \tag{15}$$

Because the distribution function is factorized in degrees of freedom u_{pnm} (i.e., the degrees of freedom are independent random quantities, moreover, with zero averages), the double sum in Eq. (15) is replaced by the single sum and the nondiagonal averages drop out. We obtain

$$\langle \xi(0, x, y) \xi(t, x', y') \rangle = 4\sum_{pmn} \langle u_{pm, 2n-1}(0) u_{pm, 2n-1}(t) \rangle \cos\left(\frac{\pi p x}{a}\right) \times \cos\left(\frac{\pi m y}{b}\right) \cos\left(\frac{\pi p x'}{a}\right) \cos\left(\frac{\pi m y'}{b}\right). \tag{16}$$

We calculate a correlator of the type $\langle u(0) u(t) \rangle$ (indices are omitted for compactness). Because the degrees of freedom satisfy the equation of motion $\ddot{u} = -\omega^2 u$, we can write $u(t)$ in the form

$$u(t) = u(0)\cos[\omega t] + \frac{\dot{u}(0)}{\omega}\sin[\omega t] \Rightarrow \langle u(0)u(t)\rangle$$
$$= \langle u^2(0)\rangle\cos[\omega t] + \langle u(0)\dot{u}(0)\rangle\frac{\sin[\omega t]}{\omega}. \tag{17}$$

The velocity $\dot{u}(0)$ in the last term in (17) is proportional to the corresponding generalized momentum, which is a random quantity independent of u. Therefore, $\langle u(0)\dot{u}(0)\rangle = 0$, and we obtain

$$\langle u(0)u(t)\rangle = \langle u^2(0)\rangle\cos[\omega t]. \tag{18}$$

By introducing the notation $\alpha \equiv M\omega^2/16kT$, we find from (12) and (13) that

$$\langle u^2(0)\rangle = \sqrt{\frac{\alpha}{\pi}}\int u^2\exp[-\alpha u^2]du$$
$$= -\sqrt{\frac{\alpha}{\pi}}\frac{\partial}{\partial\alpha}\int\exp[-\alpha u^2]du = \frac{8kT}{M\omega^2}. \tag{19}$$

Thus, by using (16), we obtain

$$\langle\xi(0, x, y)\xi(t, x', y')\rangle = \frac{32kT}{M}$$
$$\times\sum_{pmn}\frac{\cos[\omega_{pm, 2n-1}t]}{\omega^2_{pm, 2n-1}}\cos\left(\frac{\pi px}{a}\right)\cos\left(\frac{\pi my}{b}\right) \tag{20}$$
$$\times\cos\left(\frac{\pi px'}{a}\right)\cos\left(\frac{\pi my'}{b}\right).$$

Let us now take into account that we assume that the dimensions a and b are large, i.e., $a, b \gg L$. This allows us to pass from summation over p and m to integration. Introducing notations $[\pi p/a] \equiv A$ and $[\pi m/b] \equiv B$, we obtain

$$\omega_{pmn} \longrightarrow \omega_n(A, B) = v\sqrt{A^2 + B^2 + \left(\frac{\pi n}{L}\right)^2},$$
$$dA = [\pi/a], \quad dB = [\pi/b]$$

and

$$\langle\xi(0, x, y)\xi(t, x', y')\rangle = \frac{32kT}{M}\frac{ab}{\pi^2}$$
$$\times\sum_n\int_0^\infty dAdB\frac{\cos[\omega_{2n-1}(A, B)t]}{\omega^2_{2n-1}(A, B)}\cos(Ax) \tag{21}$$
$$\times\cos(By)\cos(Ax')\cos(By').$$

Because expression (4) contains correlation function (21) averaged over the light beam area, it is convenient to introduce the function $F(A, B)$, defined as

$$F(A, B) \equiv \int_{D^2}dxdy\cos[Ax]\cos[By]. \tag{22}$$

Then, taking into account that

$$\int\cos[\Omega t]e^{i\nu t}dt = \pi[\delta(\nu - \Omega) + \delta(\nu + \Omega)],$$

we obtain from (3), (21), and (22) the required spectral density $S(\nu)$ of noise in the form

$$S(\nu) = G^2(\omega)\frac{32kT}{M}\frac{ab}{\pi}$$
$$\times\sum_n^{\text{odd}}\int_0^\infty dAdB\frac{\delta[\nu - \omega_n(A, B)]}{\nu^2}F^2(A, B). \tag{23}$$

Here, the "odd" symbol shows that summation is performed over odd n. Using the presence of the delta function, we can perform integration over B to finally obtain

$$S(\nu) = \frac{G^2(\omega)}{\nu}\frac{32kT}{L\rho\pi v^2}$$
$$\times\sum_n^{\text{odd}}\int dA\left[\left(\frac{\nu}{v}\right)^2 - A^2 - \left(\frac{\pi n}{L}\right)^2\right]^{-1/2} \tag{24}$$
$$\times F^2\left(A, \sqrt{\left(\frac{\nu}{v}\right)^2 - A^2 - \left(\frac{\pi n}{L}\right)^2}\right).$$

Here, $\rho \equiv M/abL$ is the density of the interferometer material, and integration is performed in the region of variable A where the radical in (24) is real. Obviously, the contribution of the nth mode is nonzero only for $\nu > [\pi n v/L]$. As mentioned above, the maximum value of $G(\omega)$ is obtained when the interferometer is tuned to the "slope,"

$$G_{\max} = G(\omega_0 + \Delta) = \frac{I_0 Q}{2LD^2}.$$

In this case, the noise intensity is maximal,

$$S(\nu) = \frac{(I_0 Q)^2}{\nu L^3 D^4}\frac{8kT}{\rho\pi v^2}$$
$$\times\sum_n^{\text{odd}}\int dA\left[\left(\frac{\nu}{v}\right)^2 - A^2 - \left(\frac{\pi n}{L}\right)^2\right]^{-1/2} \tag{25}$$
$$\times F^2\left(A, \sqrt{\left(\frac{\nu}{v}\right)^2 - A^2 - \left(\frac{\pi n}{L}\right)^2}\right).$$

To make quantitative estimates, we can assume that the light beam has a square cross section, i.e., $x \in [a/2 - D/2, a/2 + D/2]$ and $y \in [b/2 - D/2, b/2 + D/2]$. In this case, function $F(A, B)$ (22) can be obtained in explicit form:

$$F(A, B) = \frac{4}{AB}\cos\left(\frac{Aa}{2}\right)\sin\left(\frac{AD}{2}\right)$$
$$\times\cos\left(\frac{Bb}{2}\right)\sin\left(\frac{BD}{2}\right).$$

Note that rapidly oscillating terms of the type $\cos^2[Bb/2]$ appearing in the expression for $F^2(A, B)$

can be replaced by their averages, i.e., by 1/2. Taking this into account, we can assume that

$$F^2(A, B) = \frac{1}{A^2B^2}\sin^2\left(\frac{AD}{2}\right)\sin^2\left(\frac{BD}{2}\right). \quad (26)$$

The calculations presented here are performed for the case of the "infinite lifetime" of acoustic vibrations in the resonator material, and it is for this reason that correlation function (18) does not decay. Taking into account the decay of acoustic vibrations and the parity of the correlation function in time, we obtain the correlator in the form

$$\langle u(0)u(t)\rangle = \langle u^2(0)\rangle\cos[\omega t]\exp\left(-\left|\frac{t}{\tau}\right|\right), \quad (27)$$

where τ is the decay time of acoustic modes. The noise spectrum $S_\tau(\nu)$ in this case is the convolution of (25) with a Lorentzian with a width equal to τ^{-1}.[4]

$$S_\tau(\nu) = \frac{\tau}{\pi}\int\frac{S(\nu' - \nu)}{1 + [\nu'\tau]^2}d\nu'. \quad (28)$$

3. POSSIBILITY OF OBSERVING MICRORESONATOR NOISE

The most popular optical microresonator is a Bragg resonator: a Fabry–Perot interferometer consisting of two Bragg mirrors separated by a half-wave gap. Although such microresonators are multilayer structures, the noise intensity of transmitted light can be estimated from expressions (25) and (28) for the following reason. An important feature of a simple single-layer model is that both optical and acoustic waves are localized in the same layer of a material, which is a resonator both for optical and acoustic waves. A similar situation can also take place in real Bragg resonators because an optical Bragg mirror also has an acoustic "stop band" and can efficiently reflect acoustic waves at the corresponding frequencies. In this case, the half-wave (for optical waves) gap between two Bragg mirrors can form an acoustic resonator whose properties can be approximately described by the single-layer model considered in the previous section. Quantitative estimates show that for a typical half-wave Bragg microresonator with $\lambda_0 = 2\pi c/\omega_0 = 800$ nm consisting of TiO_2 titanium oxide and SiO_2 silicon oxide layers, the frequency of the lowest acoustic mode is 10 GHz and falls within the acoustic stop band of Bragg mirrors.

Taking into account the above discussion, we will specify the following values of parameters entering expressions (25) and (28): $L = 0.276$ μm, $I_0 = 0.1$ W, $Q = 1000$, $v = 5570$ m/s (SiO_2), and $\rho = 2000$ kg/m^3 (SiO_2). For these parameter values, the frequency of the lowest acoustic mode ($n = 1$) is estimated as $\nu_1 = v/2L \approx 10$ GHz. As is seen from (25) and (28), the noise intensity increases with decreasing light spot size D. We will set $D = 10$ μm for our calculations. This spot size does not contradict the quality factor $Q = 1000$ of the transmission spectrum.[5] To estimate decay time τ of acoustic vibrations in (28), we will take into account that the quality factor of quartz resonators at frequencies of about 10^8 Hz can be on the order of 10^4–10^5. At frequencies $\sim 10^{10}$ Hz, which are of current interest here, a decrease in the quality factor of acoustic vibrations should be expected and therefore we will set in our estimates an acoustic mode quality factor an order of magnitude lower than 10^4–10^5, i.e., approximately 10^3. In this case, τ is estimated from the relation $2\pi\nu_1\tau \sim 10^3$.

To elucidate the possibility of observing the noise intensity of light transmitted through a microresonator, the noise intensity (determined by expressions (25) and (28)) should be compared with the shot noise of light:

$$S_{sn} = I_0\hbar\omega_0. \quad (29)$$

The results of such a comparison are shown in figure, which presents the noise spectra in the region of the lowest acoustic mode ($n = 1$) calculated for infinite (oscillating curve $S(\nu)$) and finite (smoothed curve $S_\tau(\nu)$) decay times τ of acoustic vibrations. The horizontal straight line shows the shot noise level (29) of the light used. Because the shot noise spectrum of light with relative fluctuations of about 1% can be recorded using modern digital spectrum analyzers[6] with an accumulation time on the order of a few seconds, there is good reason to believe that this effect can be detected even in the case when the estimates presented above are overstated by one to two orders of magnitude.

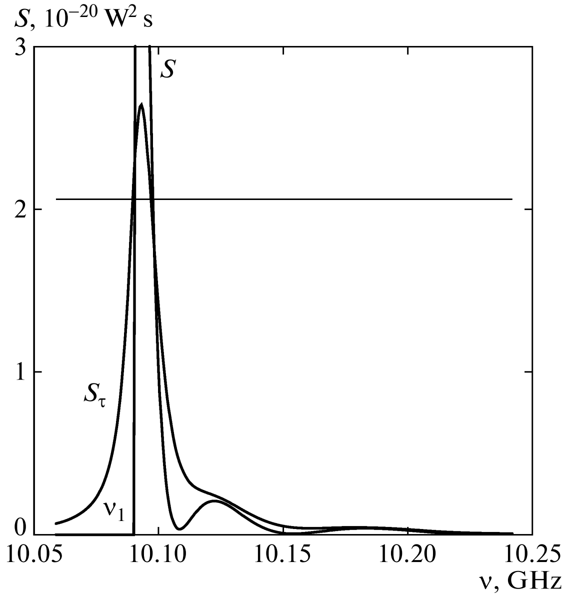
4. CONCLUSIONS

The spectral noise density of the light intensity transmitted through a microresonator has been calculated. It has been shown that the spectral density has a maximum at the frequency of acoustic vibrations of the resonator, which is similar to a Raman line. Our

[4] As we will see, the noise spectrum proves to be localized in a sufficiently narrow spectral region, where the frequency dependence of τ can be neglected.

[5] A further decrease in D by focusing can be accompanied by a decrease in the quality factor, which occurs due to an increase in the uncertainty of the angle of incidence. However, for normal incidence of a focused beam, this effect is comparatively weak because in this case the transmission coefficient of the interferometer quadratically depends on the angle of incidence.

[6] The spectral region of modern spectrum analyzers is limited by frequencies ~ 1–2 GHz. Therefore, to observe a noise signal at frequencies of ~10 GHz described in the paper, the corresponding transfer of the spectrum should be performed. A similar problem is solved in satellite TV systems by using heterodyne converts, which can be also used in the given case. In addition, the spectrum can be transferred by modulating a light beam, which takes place in mode-locked lasers.



Noise spectrum of light intensity transmitted through a microresonator. Parameter values: microresonator thickness $L = 0.276$ μm, quality factor of resonator transmission spectrum $Q = 1000$, and light beam intensity $I_0 = 0.1$ W. Calculation is performed in spectral region of lowest acoustical mode $\nu_0 \approx 10$ GHz of resonator. Oscillating curve $S(\nu)$ is the noise spectrum for infinite phonon lifetime $\tau = \infty$. Smoothed curve $S_\tau(\nu)$ is the noise spectrum for $2\pi\nu_0\tau = 2000$. The horizontal straight line is shot noise level of light.

quantitative estimates have shown that the noise appearing due to the mechanism considered in the paper can be detected using the modern noise spectroscopy technique.

As mentioned above, discussion of the informative properties of the effect described in the paper is beyond the scope of the paper, and here we present only a few brief remarks on this subject. Thermal vibrations are usually considered an interfering factor restricting the operational stability of instruments (see, e.g., [12]). Our calculation has shown that the noise spectrum of the light intensity transmitted through the microresonator is related to the spectrum of acoustic vibrations of the structure and therefore contains information similar to that obtained in Raman scattering experiments. Observation of the dependence of the noise spectrum on the light spot diameter will allow us to judge the validity of the simple model used in the paper, which neglects, e.g., the disorder of a real structure and possible localization of acoustic waves in the plane of layers. Finally, observation of the noise correlation function for two separated light beams can probably be used to estimate the radius of localized acoustic vibrations of the structure.[7]

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[7] This problem can also be solved using modern specialized Fourier analyzers.