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Spin waves in semiconductor microcavities

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We show theoretically that a weakly interacting gas of spin-polarized exciton-polaritons in a semiconductor microcavity supports propagation of spin waves. The spin waves are characterized by a parabolic dispersion at small wave vectors which is governed by the polariton-polariton interaction constant. Due to spin anisotropy of polariton-polariton interactions the dispersion of spin waves depends on the orientation of the total polariton spin. For the same reason, the frequency of homogeneous spin precession/polariton spin resonance depends on their polarization degree.

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Introduction. Spin waves are weakly damped harmonic oscillations of spin polarization. Predicted to appear in Fermi liquids over 50 years ago [1], and discovered at the end of the 1960s in metals [2], they are among the most fascinating manifestations of collective effects in interacting systems. Later it was understood theoretically that the spin waves can exist in a nondegenerate electron gas [3] as well as in atomic gases [4,5], and the spin waves were indeed observed in a number of interacting gases such as hydrogen and helium [6–8] (see Ref. [9] for a review). Spin waves were also observed in atomic Bose gases, namely, in ^{87}Rb vapors at temperatures of about 850 nK, which exceeded the Bose-Einstein condensation temperature [10] (see also Refs. [11–13] where this experiment was interpreted).

Recently, semiconductor microcavities with quantum wells sandwiched between highly reflective mirrors have attracted a lot of interest in the solid state and photonics communities [14]. In these artificial structures the strong coupling is achieved between excitons, being material excitations in quantum wells, and photons confined between the mirrors [15]. Resulting mixed light-matter particles, exciton-polaritons, demonstrate the Bose-Einstein statistics and may condense at critical temperatures ranging from tens of Kelvin [16] to several hundred Kelvin [17,18], which exceeds by many orders of magnitude the Bose-Einstein condensation temperature in atomic gases. High transition temperatures and the strong coupling with light makes semiconductor microcavities perfectly suited for benchtop studies of collective effects of bosons.

In typical GaAs based microcavities, exciton-polaritons may have two spin projections onto the structure growth axis, ± 1, corresponding to right- and left- circular polarizations of photons (and spin moment of excitons) forming polaritons. Owing to the composite nature of exciton-polaritons, the interactions between them are strongly spin dependent [14,15,19]. A number of prominent spin-related phenomena both in interacting and in noninteracting polariton systems have already been predicted and observed in the microcavities, such as, e.g., polarization multistability [20,21] and optical spin Hall effect [22,23] (see Refs. [14,15,19] for reviews).

Here we predict the existence of weakly damped spin waves for a nondegenerate or weakly degenerate polariton gas in a microcavity with embedded quantum wells. We show that the system sustains the spin wave solutions, where the spin of polaritons $\boldsymbol{S}$ is harmonic function of the coordinate $\boldsymbol{r}$ and time t, $\boldsymbol{S} \propto \exp(i\boldsymbol{q}\boldsymbol{r} - i\omega t)$, and calculate their dispersion, $\omega \equiv \omega(\boldsymbol{q})$. The stability of spin waves is analyzed. The experimental manifestations of spin waves in the photoluminescence spectroscopy and spin noise studies are discussed. Due to the strong light-matter interaction in microcavities, which allows one to observe directly the spin states of quasiparticles, microcavities may become one of the most suitable systems for spin waves experimental studies.

Model. We consider nondegenerate or weakly degenerate polariton gas at a temperature higher than the Berezinskii-Kosterlitz-Thouless transition temperature with weak interactions; in this case the single-particle spin density matrix is parametrized as $\hat{\rho}_{\boldsymbol{k}} = (N_{\boldsymbol{k}}/2)\hat{I} + \boldsymbol{S}_{\boldsymbol{k}} \cdot \hat{\boldsymbol{\sigma}}$, where $N_{\boldsymbol{k}}$ is the occupancy of the orbital state with the wave vector $\boldsymbol{k}$, $\boldsymbol{S}_{\boldsymbol{k}} \equiv \boldsymbol{S}_{\boldsymbol{k}}(\boldsymbol{r})$ is the coordinate $\boldsymbol{r}$-dependent spin distribution function, and $\hat{I}$ and $\hat{\boldsymbol{\sigma}}$ are 2×2 unit and Pauli matrices, respectively. The spin distribution function satisfies the kinetic equation, which describes the rate of $\boldsymbol{S}_{\boldsymbol{k}}$ change in time as a result of the particle propagation with the group velocity $\boldsymbol{v}_{\boldsymbol{k}}$, spin precession in the effective field $\boldsymbol{\Omega}_{\boldsymbol{k}}^{(\mathrm{eff})}$, as well as generation and scattering processes described by the collision integral $\boldsymbol{Q}\{\boldsymbol{S}_{\boldsymbol{k}}\}$ [24]:

$$\frac{\partial \boldsymbol{S}_{\boldsymbol{k}}}{\partial t} + \boldsymbol{v}_{\boldsymbol{k}} \cdot \frac{\partial \boldsymbol{S}_{\boldsymbol{k}}}{\partial \boldsymbol{r}} + \boldsymbol{S}_{\boldsymbol{k}} \times \boldsymbol{\Omega}_{\boldsymbol{k}}^{(\mathrm{eff})} = \boldsymbol{Q}\{\boldsymbol{S}_{\boldsymbol{k}}\}. \quad (1)$$

Here

$$\boldsymbol{\Omega}_{\boldsymbol{k}}^{(\mathrm{eff})} = \alpha_1 \sum_{\boldsymbol{k}'} S_{\boldsymbol{k}',z}\boldsymbol{e}_z + \boldsymbol{\Omega}_L, \quad (2)$$

where the constant α_1 describes the interaction of polaritons with parallel spins. We recall that the polariton-polariton interactions are strongly spin anisotropic and neglect weak interaction of particles with opposite signs of circular polarization [25–28]. The effective field $\boldsymbol{\Omega}_L = \hbar^{-1} g\mu_B B_z \boldsymbol{e}_z + \Omega_a \boldsymbol{e}_x + \boldsymbol{\Omega}(\boldsymbol{k})$, where $\boldsymbol{e}_i$ are unit vectors of Cartesian axes, $i = x,y,z$, is interaction independent; generally, it is contributed by an external magnetic field $\boldsymbol{B}$, g is the exciton-polariton g-factor [29], the splitting of linearly polarized polariton states due to the structure anisotropy, Ω_a (the anisotropy field is parallel to the x axis), and transverse-electric–transverse-magnetic

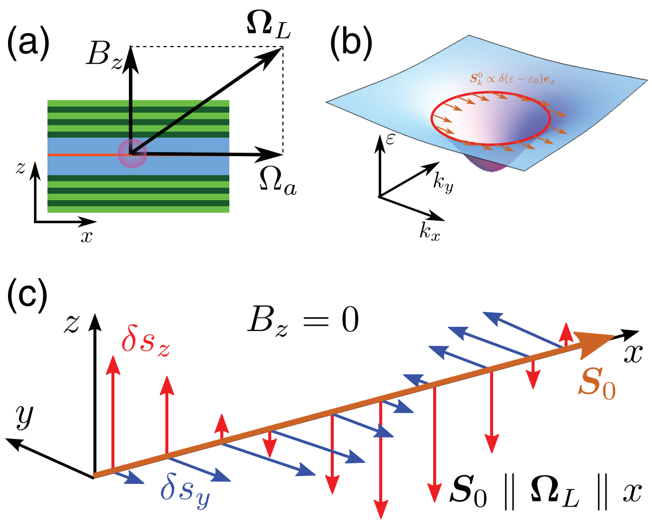

FIG. 1. (Color online) (a) Schematics of the microcavity structure and external fields acting on the polariton pseudospin. (b) Sketch of polariton dispersion (surface) and elastic circle (red circle). The distribution function of polariton spin in the case of linearly polarized excitation of the states on the elastic circle is illustrated by arrows. (c) Schematic illustration of the spin wave propagating along the x axis at $B_z = 0$; red and blue arrows demonstrate δs_z and δs_y components.

(TE-TM) splitting of the cavity modes, $\boldsymbol{\Omega}(\boldsymbol{k})$. We assume that the structure anisotropy is strong enough, $\Omega_a \gg \Omega(\boldsymbol{k})$ for the relevant wave vectors range and neglect TE-TM splitting [30]. Under our assumptions the effective field is independent of $\boldsymbol{k}$ and acts similarly to the real magnetic field. Depending on the ratio of $g\mu_B B_z/\hbar$ and Ω_a this field can be arbitrarily oriented [see Fig. 1(a)]. The collision integral in the right-hand side of Eq. (1) accounts for the polariton generation, scattering, and decay processes,

$$\boldsymbol{Q}\{\boldsymbol{S}_{\boldsymbol{k}}\} = -\frac{\boldsymbol{S}_{\boldsymbol{k}}}{\tau_0} + \boldsymbol{g}_{\boldsymbol{k}} + \sum_{\boldsymbol{k}'}[W_{\boldsymbol{k}\boldsymbol{k}'}\boldsymbol{S}_{\boldsymbol{k}'} - W_{\boldsymbol{k}'\boldsymbol{k}}\boldsymbol{S}_{\boldsymbol{k}}]. \quad (3)$$

Here τ_0 is the lifetime of polaritons, $\boldsymbol{g}_{\boldsymbol{k}}$ is the polariton generation rate accounting for the incoming flow of quasi-particles from the reservoir, and $W_{\boldsymbol{k}\boldsymbol{k}'}$ is the scattering rate from the state $\boldsymbol{k}$ to the state $\boldsymbol{k}'$ which accounts for both elastic and inelastic scattering processes. Due to the bosonic nature of exciton-polaritons and polariton-polariton interactions, $\boldsymbol{g}_{\boldsymbol{k}}$ and $W_{\boldsymbol{k}\boldsymbol{k}'}$ depend, generally, on the occupancies and spin polarizations in the states $\boldsymbol{k}$,$\boldsymbol{k}'$ [24,25,33]. The dynamics of polaritons can be described by Eq. (1) which is valid provided that the renormalization of spectrum due to polariton-polariton interactions is negligible, otherwise the excitation spectrum should be found from the spin-dependent Gross-Pitaevskii equation [34–36].

Under the steady-state excitation, the quasiequilibrium distribution $\boldsymbol{S}_k^{(0)}$ of exciton-polaritons is formed whose shape is determined by the generation, thermalization, and interactions. This function satisfies Eq. (1) with derivatives $\partial/\partial t, \partial/\partial \boldsymbol{r}$ being equal to zero. For simplicity we assume that the pumping is isotropic, hence $\boldsymbol{S}_k^{(0)}$ depends only on the absolute value of the polariton wave vector $k = |\boldsymbol{k}|$. Its specific form is determined by the pumping conditions. In what follows, two important limiting cases are addressed: (i) quasiresonant pumping which creates monoenergetic polaritons (so-called excitation of elastic circle); such a situation can be experimentally realized if the pump energy is slightly above the inflection point on the dispersion Fig. 1(b), so that the phonon-assisted relaxation towards the ground state is suppressed due to the polaritons strong energy dispersion (bottleneck effect), and (ii) nonresonant pumping which creates thermalized distribution of particles. In the case of quasiresonant excitation of polaritons with the same energy ε_0 by a polarized light, the spin distribution function $\boldsymbol{S}_k^{(0)} \propto \delta(\varepsilon - \varepsilon_0)\boldsymbol{e}_i$, where $i = x$ or y for the linearly polarized excitation and $i = z$ for the circularly polarized one, $\varepsilon \equiv \varepsilon_k$ is the polariton dispersion [see scheme in Fig. 1(b)], while for thermalized polaritons (at the temperature T higher than the degeneracy temperature), $\boldsymbol{S}_k^{(0)} \propto \boldsymbol{S}_0 \exp(-\varepsilon/T)$ with the prefactor $\boldsymbol{S}_0$ dependent on the effective field $\boldsymbol{\Omega}_L$. In order to analyze the spin excitations, the total spin distribution function is presented as a sum of its quasiequilibrium part $\boldsymbol{S}_k^{(0)}$ and the fluctuating correction $\delta \boldsymbol{s}_{\boldsymbol{k}} \ll \boldsymbol{S}_{\boldsymbol{k}}^{(0)}$. A standard linearization of Eq. (1) and substitution of $\delta \boldsymbol{s}_{\boldsymbol{k}} = \exp(i\boldsymbol{q}\boldsymbol{r} - i\omega t)\boldsymbol{s}_{\boldsymbol{k}}$ with $\boldsymbol{q}$ being the wave vector and ω being the frequency of the fluctuation yields (cf. [1,9])

$$\left[\tau_c^{-1} - i\omega + i(\boldsymbol{q}\cdot\boldsymbol{v}_{\boldsymbol{k}})\right]\boldsymbol{s}_{\boldsymbol{k}} + \alpha_1 \boldsymbol{S}_k^{(0)} \times \boldsymbol{e}_z \sum_{\boldsymbol{k}'} s_{\boldsymbol{k}',z} + \boldsymbol{s}_{\boldsymbol{k}} \times \left(\boldsymbol{\Omega}_L + \alpha_1 \boldsymbol{e}_z \sum_{\boldsymbol{k}'} S_{\boldsymbol{k}',z}^{(0)}\right) = -\frac{\boldsymbol{s}_{\boldsymbol{k}} - \bar{\boldsymbol{s}}_{\boldsymbol{k}}}{\tau}, \quad (4)$$

where we introduced the lifetime of the fluctuation τ_c and the isotropization time τ. The overbar symbolizes averaging over possible orientations of $\boldsymbol{k}$. In the simplest approximation, polaritons are assumed to be supplied directly by the polarized pump or from the incoherent but spin-polarized reservoir, the polariton-polariton scattering is neglected as well as inelastic processes, and the elastic scattering is assumed to be isotropic, in which case $W_{\boldsymbol{k},\boldsymbol{k}'} = W(\varepsilon_k)\delta(\varepsilon_k - \varepsilon_{k'})$, $\tau^{-1} = \sum_{\boldsymbol{k}'} W(\varepsilon_k)\delta(\varepsilon_k - \varepsilon_{k'})$. The lifetime of a fluctuation is governed by an interplay of polariton decay processes accounted for by the lifetime τ_0 in our formalism, and by the bosonic stimulation effect, which increases the lifetime of the fluctuations, $\tau_c = \tau_0(1 + N_k)$ [24]. Equation (4) determines the dynamics of spin fluctuations in the system. Its eigenmodes represent the spin waves in the interacting polariton ensemble.

Results. The solution of Eq. (4) can be expressed by decomposing the function $\boldsymbol{s}_{\boldsymbol{k}}$ in the angular harmonics of the polariton wave vector $\boldsymbol{k}$ as $\boldsymbol{s}_{\boldsymbol{k}} = \sum_m \exp(im\varphi)\boldsymbol{s}_m(\varepsilon)$, with φ being the azimuthal angle of $\boldsymbol{k}$, and reducing Eq. (4) to a system of equations for the energy-dependent functions $\boldsymbol{s}_m(\varepsilon)$. The condition of compatibility for this system of equations yields dispersions of the waves. Below we analyze the spectrum of excitations and eigenmodes for different particular cases.

Homogeneous excitations. We start the analysis from the homogeneous case, $\boldsymbol{q} = 0$. In this case the angular harmonics $\exp(im\varphi)\boldsymbol{s}_m(\varepsilon)$ are the eigensolutions of Eq. (4). For all $m \neq 0$ one eigenmode corresponds to the damped solution with $\boldsymbol{s}_m$ parallel to the total field $\boldsymbol{\Omega}^{(\text{tot})} = \boldsymbol{\Omega}_L + \alpha_1 S_{0,z}\boldsymbol{e}_z$, $\boldsymbol{S}_0 = \sum_{\boldsymbol{k}'} \boldsymbol{S}_{k'}^0$, whose damping rate is $\nu = \tau_c^{-1} + \tau^{-1}$, and two

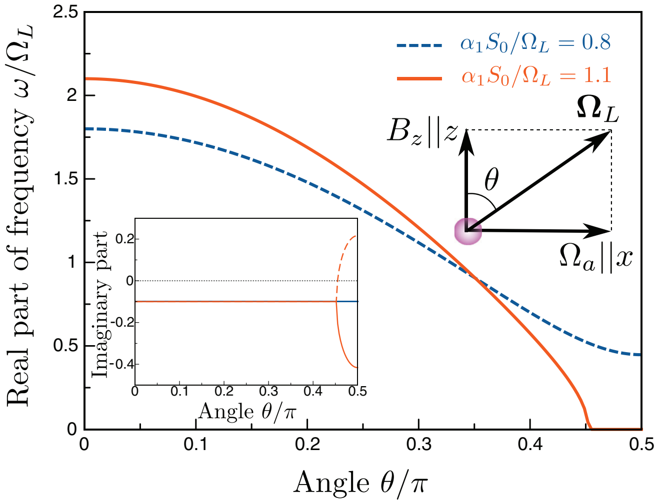

FIG. 2. (Color online) Eigenfrequencies of homogeneous precessing modes with $m = 0$ as a function of angle θ calculated after Eq. (6). The main panel shows the real part of the frequencies and the inset shows the imaginary parts. The parameters of calculation are $\alpha_1 S_0 \Omega_L = 0.8$ (blue/dashed) and 1.1 (red/solid), $\Omega_L \tau_c = 10$.

other eigenmodes precessing in the plane perpendicular to $\boldsymbol{\Omega}^{(\mathrm{tot})}$ with frequencies $\Omega^{(\mathrm{tot})}$ and the damping rate ν.

The harmonic with $m = 0$ is isotropic in $\boldsymbol{k}$ space; its eigenfrequency corresponds to the spin resonance frequency. We introduce $\tilde{\boldsymbol{s}}_0 = \sum_{\boldsymbol{k}} \boldsymbol{s}_0(\varepsilon)$ and perform the summation of Eq. (4) over $\boldsymbol{k}$ which yields

$$\left(\tau_c^{-1} - i\omega\right)\tilde{\boldsymbol{s}}_0 - \alpha_1 \tilde{s}_{0,z}\boldsymbol{e}_z \times \boldsymbol{S}_0 + \alpha_1 \tilde{\boldsymbol{s}}_0 \times \boldsymbol{e}_z S_{0,z} + \tilde{\boldsymbol{s}}_0 \times \boldsymbol{\Omega}_L = 0. \tag{5}$$

We recall that in the case of spin-anisotropic interactions the Larmor theorem [37] is not applicable, and the homogeneous spin excitation frequency can be renormalized by the interactions. To illustrate it we consider the dependence of the spin resonance frequency on the orientation of effective Larmor field $\boldsymbol{\Omega}_L$ and spin polarization $\boldsymbol{S}(0)$. The orientation of $\boldsymbol{\Omega}_L$ in the (xz) plane can be varied by changing the external magnetic field contributing to $\Omega_{L,z}$ or mechanical strain contributing to $\Omega_{L,x} = \Omega_a$. We assume efficient thermalization in the spin space, $\boldsymbol{S}_0 \parallel \boldsymbol{\Omega}_L$, introduce the angle θ between $\boldsymbol{S}_0$ and the z axis, and present the complex eigenfrequencies of Eq. (5) in a form [28]

$$\omega_0 = -\frac{i}{\tau_c},$$
$$\omega_\pm = -\frac{i}{\tau_c} \pm \sqrt{\Omega_L^2 + \alpha_1^2 S_0^2 \cos^2\theta + \alpha_1 \Omega_L S_0 (3\cos^2\theta - 1)}. \tag{6}$$

For instance, if $\boldsymbol{\Omega}_L$ and $\boldsymbol{S}_0$ are parallel to the z axis, $\theta = 0$, the frequencies of the precessing modes are $\pm|\Omega_{L,z} + \alpha_1 S_{0,z}|$ and the damping rate is $1/\tau_c$.

The real part of ω_+ and imaginary parts of $\omega_\pm$ are shown in Fig. 2 as a function of the angle between the field and the z axis θ. Depending on the sign $\alpha_1 S_0 \Omega_L$ the frequency can increase or decrease with an increase of θ. Note, that for large enough $\alpha_1 S_0$, $\omega_\pm$ become imaginary as shown by the red solid curve in Fig. 2. Moreover, the imaginary part of one of the frequencies can be positive, which manifests the instability of the system (see inset in Fig. 2). To analyze it in more detail we put $\theta = \pi/2$ ($B_z = 0$, $\boldsymbol{S}_0$ and $\boldsymbol{\Omega}_L$ are in the structure plane). In this case $\omega_\pm = -i/\tau_c \pm \sqrt{\Omega_L^2 - \alpha_1 S_0 \Omega_L}$. The system becomes unstable for

$$\alpha_1 S_0 \Omega_L > \Omega_L^2 + \frac{1}{\tau_c^2}, \tag{7}$$

and small fluctuations of s_y and s_z grow exponentially. This is because the anisotropic interactions between polaritons favor in-plane orientation of the pseudospin [19,20]. The instability of small spin fluctuations can result in the nonlinear oscillations of spin polarization [31] similar to those discussed in Refs. [38,39] or in changes in the polarization of the ground state accompanied by change of orientation of $\boldsymbol{S}_0$ (cf. [40]) where condition (7) no longer holds.

Spin waves. Let us consider the spatially inhomogeneous solutions of Eq. (4) which describe the propagation of spin fluctuations and spin waves. To be specific, we consider the case where $\boldsymbol{S}_k^0$ and $\boldsymbol{\Omega}_L$ are parallel to the x axis, and to simplify the treatment we assume $\tau \gg \tau_c$ [28]. Moreover, we assume that the system is stable at $\boldsymbol{q} = 0$, i.e., the condition (7) is not fulfilled. We seek the solution, which corresponds to the precessing mode at $\boldsymbol{q} = 0$, where $s_{k,x} = 0$. From Eq. (4) we arrive at the set of linear homogeneous integral equations for $s_{\boldsymbol{k},y}$ and $s_{\boldsymbol{k},z}$, whose self-consistency requirement yields

$$\sum_{\boldsymbol{k}} \frac{\alpha_1 \Omega_L \tau_c^2 S_k^{(0)}}{[1 - i\omega\tau_c + i(\boldsymbol{q}\boldsymbol{v}_{\boldsymbol{k}})\tau_c]^2 + (\Omega_L \tau_c)^2} = 1. \tag{8}$$

This equation describes the dispersion of spin waves. It has a more complex form compared with the dispersion equation for the spin waves in the systems with spin-isotropic interactions [1,3,9].

To solve Eq. (8) one has to specify the function $S_k^{(0)}$ whose form is determined by the excitation conditions. First, we consider the case of resonant excitation of polaritons at the elastic circle where a monoenergetic distribution of particles is generated. For the isotropic dispersion the product $\boldsymbol{q}\boldsymbol{v}_{\boldsymbol{k}} = qv_0\cos\varphi$, $v_0 = \hbar^{-1} d\varepsilon_k/dk$ is the polariton velocity on the elastic circle. Here the summation over $\boldsymbol{k}$ reduces to the averaging over the azimuthal angle φ and Eq. (8) takes the form

$$\frac{1}{\sqrt{\tilde{\omega}_+^2 - (qv_0)^2}} - \frac{1}{\sqrt{\tilde{\omega}_-^2 - (qv_0)^2}} = \frac{2}{\alpha_1 S_0}, \tag{9}$$

where $\tilde{\omega}_\pm = \omega \pm \Omega_L - i/\tau_c$. Equation (9) determines the dispersion of the spin waves. For $q = 0$ it passes to $\omega_\pm$ in Eq. (6). For small $qv_0 \ll |\alpha_1 S_0|$ and $|\alpha_1 S_0| \ll |\Omega_L|$ the dispersion of the spin waves reads

$$\omega(q) = \Omega_L - \alpha_1 S_0/2 - (qv_0)^2/(\alpha_1 S_0) - i/\tau_c, \tag{10}$$

where we took the solution which passes to ω_+ in Eq. (6). As follows from Eq. (10) the dispersion is parabolic for small qv_0 and the "effective mass" is proportional to α_1. Similarly to the previously studied electronic and atomic systems [1,3,9,41] the dispersion of spin wave results from an interplay of the gradient, $\propto \boldsymbol{q}\boldsymbol{v}_{\boldsymbol{k}}$, and interaction, $\propto \alpha_1$, terms in the kinetic equation (4). Indeed, owing to the gradient contribution, the

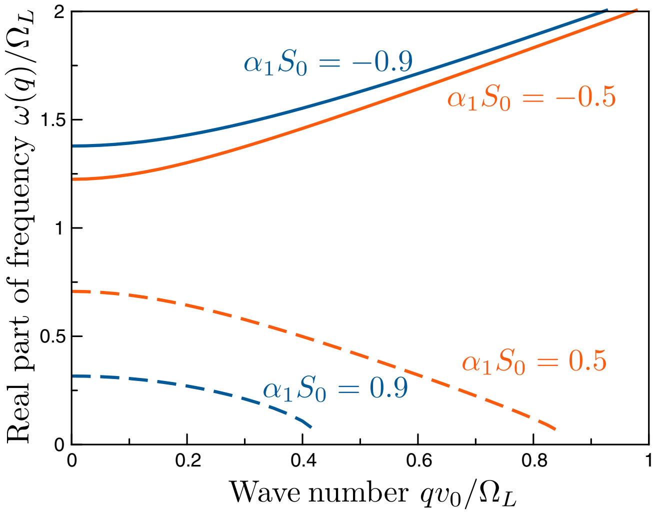

FIG. 3. (Color online) Dispersion of spin waves in the case of resonant excitation calculated after Eq. (9). The parameters of calculation are indicated at each curve.

spin density in $\boldsymbol{k}$ space acquires $\propto (qv_0)^2$ correction, yielding gain or loss of energy depending on the sign of $\alpha_1 S_0$. The spin wave frequency increases with the increase of the wave vector for $\alpha_1 S_0 < 0$ and decrease for $\alpha_1 S_0 > 0$. The behavior of $\omega(q)$ is illustrated in Fig. 3. Noteworthy, for the solutions with $\alpha_1 S_0 > 0$, the real part of the frequency vanishes at some q, in which case the solutions of Eq. (10) may become unstable. For arbitrary direction of $\boldsymbol{\Omega}_L$ and $\boldsymbol{S}_0$ the dispersion of waves has a similar form, but its parameters depend on the orientation of the magnetic field and the total spin due to anisotropy of polariton-polariton interactions.

It is instructive to compare the parabolic dispersion of spin waves in a weakly interacting polariton gas with the dispersion of excitations of an interacting polariton condensate [34–36]. In the case of a condensate of polaritons (in the absence of TE-TM splitting) the dispersion of excitations is linear. By contrast, the dispersion of spin waves for noncondensed polaritons is parabolic at small wave vectors.

In the case of a nonresonant excitation where a continuous distribution $S_k^{(0)}$ is formed, the analysis of the dispersion of spin waves is more complex, particularly because the additional channel of damping caused by the spatial dispersion appears [3,9], but the basic physics remains the same. To illustrate this we consider the case of a thermalized nondegenerate gas where $S_k^{(0)}$ is described by the Boltzmann function characterized by an effective temperature T. The evaluation of the sum in Eq. (8) under the assumptions $|\alpha_1 S_0| \ll |\Omega_L|$, $|\alpha_1 S_0|\tau_c \gg 1$, and $qv_T \ll |\alpha_2 S_0|$, where $v_T = \sqrt{k_B T/m}$ is the thermal velocity with m being the polariton effective mass and k_B being the Boltzmann constant, and the solution of the resulting equation yields the dispersion in the form

$$\omega(q) = \Omega_L - \alpha_1 S_0/2 - \frac{2(qv_T)^2}{\alpha_1 S_0} - i\gamma_L, \qquad (11)$$

where the Landau damping can be estimated as

$$\gamma_L = \sqrt{\frac{\pi}{2}}\frac{(\alpha_1 S_0)^2}{4qv_T}\exp\left(-\frac{(\alpha_1 S_0)^2}{8(qv_T)^2} - 1\right), \qquad (12)$$

and it is exponentially small for small wave vectors, in agreement with Refs. [3,9]. The allowance for Landau damping in Eq. (11) is correct only if the damping is large enough compared with $1/\tau_c$ but small compared to $|\alpha_1 S_0|$.

Conclusions. To conclude, we predicted the existence of exciton-polariton spin waves in semiconductor microcavities with embedded quantum wells. The dispersion and damping of spin waves were calculated in two important particular cases: (i) resonant excitation of a quasimonoenergetic distribution of polaritons at an elastic circle and (ii) nonresonant excitation, where the Boltzmann distribution of quasiparticles is formed. In the state-of-the-art microcavities the polariton polarization splittings induced by the cavity anisotropy, $\hbar\Omega_L$, and interaction-induced effective field $\hbar\alpha_1 S_0$ are of the order of 100 μeV, usually [42–44]. For the polariton lifetime $\gtrsim$10 ps the spin waves can be readily detectable even at the relatively weak pump. The spin waves can be excited, e.g., in two-beam photoluminescence experiments where the continuous-wave beam creates the desired steady distribution of polaritons with a given spin polarization $\boldsymbol{S}_k^{(0)}$ and the probe beam injects a small nonequilibrium portion of polaritons with the spin polarization different from $\boldsymbol{S}_k^{(0)}$. The time-resolved microphotoluminescence spectroscopy as used, e.g., in Refs. [45,46] would be a suitable tool for detection of the spin waves. Another possibility to observe the spin waves is to use the spin noise spectroscopy [47,48] and measure temporal and spatial correlations of spin fluctuations in the presence of the pump only [31].

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[28] Results for the general case will be presented elsewhere.
[29] The Zeeman effect caused by the in-plane components of the external field is negligible since it mixes dark and bright excitonic states.
[30] The TE-TM splitting itself results in the nontrivial spin dynamics of polaritons [28,31]. It can be also suppressed by design of microcavity [32].
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Supplementary materials to:
Spin waves in quantum microcavities

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I. INTERACTION-INDUCED INSTABILITY OF POLARITON SPIN DYNAMICS

The linear analysis of kinetic equation (5) in the main text shows that the polariton spin system becomes unstable at $\theta = \pi/2$ ($\boldsymbol{S}_0 \parallel \boldsymbol{\Omega}_L$ is in the structure plane) provided that

$$\alpha_1 S_0 \Omega_L > \Omega_L^2 + \frac{1}{\tau_c^2}, \tag{1}$$

see Eq. (7) of the main text. In the linear regime the excitations are grow exponentially (in the unstable regime) with the increment

$$\lambda = \sqrt{\alpha_1 S_0 \Omega_L - \Omega_L^2} - 1/\tau_c \tag{2}$$

In order to analyze the instability in more detail we consider the simplest possible case where polariton generation and dissipation are absent and the spin dynamics is described by the following equation

$$\frac{\partial \boldsymbol{S}}{\partial t} + \boldsymbol{S} \times \boldsymbol{\Omega}^{(\mathrm{eff})} = 0, \tag{3}$$

with

$$\boldsymbol{\Omega}_{\boldsymbol{k}}^{(\mathrm{eff})} = \boldsymbol{\Omega}_L + \alpha_1 \sum_{\boldsymbol{k}'} S_{\boldsymbol{k}',z} \boldsymbol{e}_z, \tag{4}$$

see Eqs. (1) and (2) of the main text.

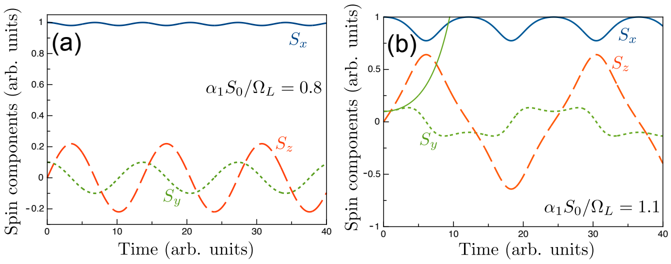

Figure 1: Panels (a) and (b) show temporal dynamics of the total spin components nonlinear Eqs. (3), (4) for homogeneous case for $\boldsymbol{\Omega}_L \parallel \boldsymbol{S}_0 || x$ (at $t = 0$) and small $\delta s_y(0) = S_0/10$. (a) corresponds to stable and (b) to unstable regimes. Spin components are marked at each curve, thin solid line shows exponentially growing solution calculated after Eq. (2).

Examples of temporal dynamics of the total polariton pseudospin in the stable and unstable regimes calculated numerically after Eq. (3) are shown in Fig. 1, panels (a) and (b), respectively. It is seen that the instability of small spin fluctuations can result in the nonlinear oscillations of spin polarization similar to those discussed in Refs. [1, 2] [Fig. 1(b)]. The detailed analysis of the final state of the system and its stability with allowance for dissipative processes is beyond the scope of the present paper.

II. SPIN PRECESSION CAUSED BY TE-TM SPLITTING

Here we demonstrate that the TE-TM splitting of the polariton states can also result in the weakly damped spin resonance and diffusive spin modes. We consider symmetric cavity with the TE-TM splitting $\boldsymbol{\Omega}_{\boldsymbol{k}}$ in the form

$$\boldsymbol{\Omega}_{\boldsymbol{k}} = \Omega(k)[\cos\varphi, \sin\varphi, 0], \tag{5}$$

where φ is the angle between $\boldsymbol{k}$ and x-axis in the structure plane, $\Omega(k)$ is the amplitude of the TE-TM splitting. For simplicity we consider monoenergetic polaritons with the energy $\varepsilon_0 \equiv \varepsilon(k_0)$, introduce $\Omega_0 = \Omega(k_0)$, and assume that both $\Omega_0\tau_c, \Omega_0\tau \gg 1$. We focus on dynamics of spin z component. For $\boldsymbol{q} = 0$ we obtain (note that S_z is isotropic function of φ, while S_x and S_y are strongly anisotropic and their angular averages are 0):

$$\left(-\mathrm{i}\omega + \frac{1}{\tau_c}\right) S_{k,z} + S_{\boldsymbol{k},x}\Omega_{0,y} - S_{\boldsymbol{k},y}\Omega_{0,x} = 0, \tag{6a}$$

$$\left(-\mathrm{i}\omega + \frac{1}{\tau_c} + \frac{1}{\tau}\right) S_{\boldsymbol{k},x} - S_{k,z}\Omega_{0,y} = 0, \tag{6b}$$

$$\left(-\mathrm{i}\omega + \frac{1}{\tau_c} + \frac{1}{\tau}\right) S_{\boldsymbol{k},y} + S_{k,z}\Omega_{0,x} = 0. \tag{6c}$$

Solution of Eqs. (6) yields the frequency of homogeneous spin z component oscillations

$$\omega = \Omega_0\sqrt{1 - [2\Omega_0\tau]^{-2}} - \frac{\mathrm{i}}{\tau_c} - \frac{\mathrm{i}}{2\tau}. \tag{7}$$

In the relevant limiting case $\Omega(k)\tau \gg 1$ spin z component demonstrates oscillations with the frequency $\Omega(k)$ similarly to the oscillations of spin in high-mobility electron gas [3–5]. Note that for $\Omega(k)\tau \ll 1$ the frequency is purely imaginary, $\omega = -\mathrm{i}\Omega_0^2\tau - \mathrm{i}/\tau_c$.

The spectrum of inhomogeneous spin excitations can be conveniently found in the limit of $qv_0\tau \ll 1$, in which case the gradient term $\propto (\boldsymbol{q}\boldsymbol{v}_{\boldsymbol{k}})$, Eqs. (1) and (4) of the main text, can be taken into account by perturbation theory. After some algebra we obtain for monoenergetic particles in the limit of $\Omega_0\tau_c, \Omega_0\tau \gg 1$, $qv_0\tau \ll 1$:

$$\omega(q) = \Omega_0 - \frac{\mathrm{i}}{\tau_c} - \frac{\mathrm{i}}{2\tau} - \mathrm{i}(qv_0)^2\tau. \tag{8}$$

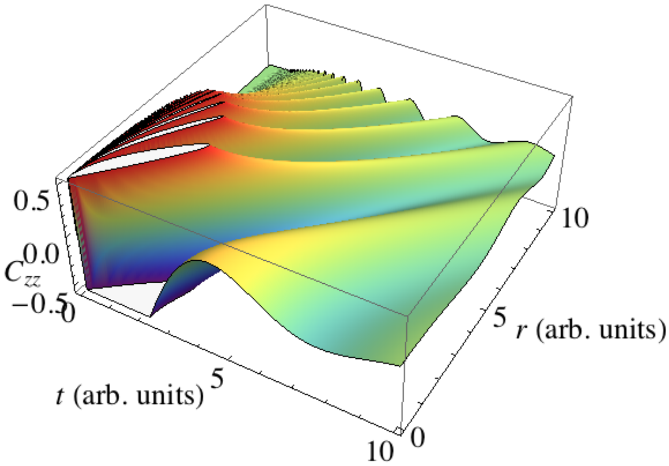

Figure 2: Correlator of polariton spins $C_{zz} = \langle \delta S_z(r,t)\delta S_z(0,0)\rangle$ calculated after Eq. (11). The calculation parameters are as follows: $\Omega = 1$, $\tau_c = 10$, $\mathcal{A}' = 0.25$, $\mathcal{A}'' = 0.01$ (coordinates and time are given in dimensionless units).

Unlike the spin waves predicted in the main text, Eq. (10), here the spatial inhomogeneity results in the diffusive damping of the spin precession mode. The analysis of the interplay between interactions and TE-TM splitting is beyond the present paper.

III. SPATIAL CORRELATIONS OF POLARITON SPINS

Spin waves describe the time-space correlations of polariton spins and can be addressed in the two-beam spin noise spectroscopy technique (for reviews on the spin noise spectroscopy see, e.g., Refs. [6, 7], temporal and spatial fluctuations of spin density were studied for electrons for the first time in Refs. [8, 9], respectively). The correlation function of polariton spin fluctuations can be expressed via the spin waves spectrum $\omega(q)$ as

$$\langle \delta S_z(\boldsymbol{r},t)\delta S_z(0,0)\rangle \propto \int \frac{\mathrm{d}\omega}{2\pi}\sum_{\boldsymbol{q}} \frac{\exp\left(\mathrm{i}\boldsymbol{qr}-\mathrm{i}\omega t\right)}{\omega-\omega(q)}. \quad (9)$$

In the long-wavelength limit [see Eqs. (10) and (11) of the main text]

$$\omega(q) = \Omega - \mathcal{A}q^2 - \mathrm{i}/\tau_c, \quad (10)$$

where Ω and $\mathcal{A}$ are the parameters depending on the effective field, $\boldsymbol{S}_0$ and interactions. Note that diffusion of particles yields imaginary part $\mathcal{A}''$ of $\mathcal{A} = \mathcal{A}' + \mathrm{i}\mathcal{A}''$ in addition to its real part $\mathcal{A}'$ determined by interactions. Making use of Eq. (10) we arrive from Eq. (9) to

$$\langle \delta S_z(r,t)\delta S_z(0,0)\rangle = \mathrm{Im}\left\{\frac{\exp\left[\mathrm{i}\left(\frac{r^2}{4\mathcal{A}t}-\Omega t\right)\right]}{4\pi\mathcal{A}t}\right\}\mathrm{e}^{-t/\tau_c}. \quad (11)$$

The spin correlation function is presented in Fig. 2. The oscillations of the correlation function as a function of coordinate and time are clearly seen demonstrating wave-like propagation of spin excitations.

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